Simulations of continuous nuclear demagnetization refrigerators

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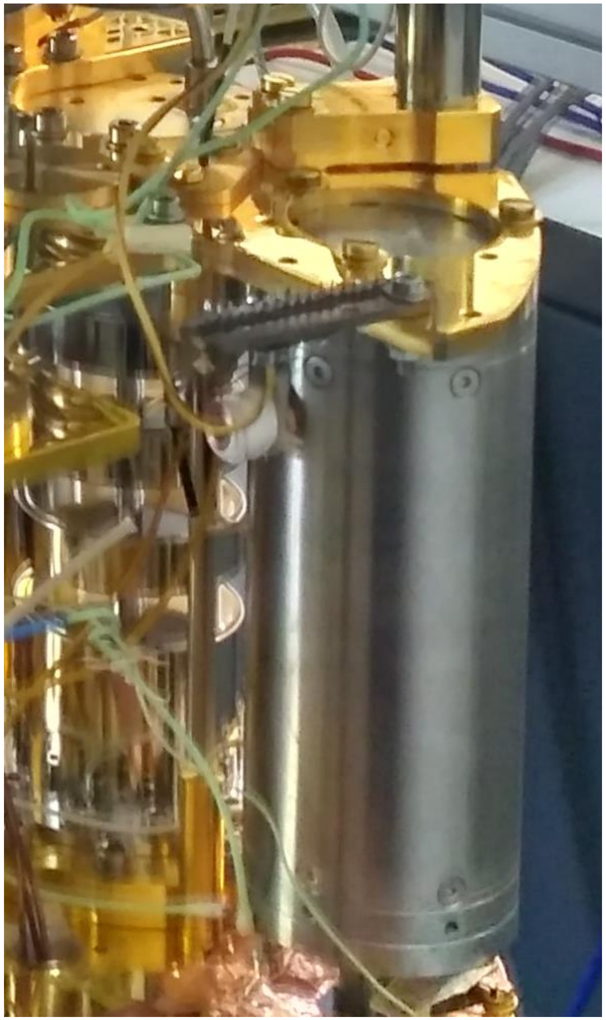
Motivation



Continuous sub-mK cooling:

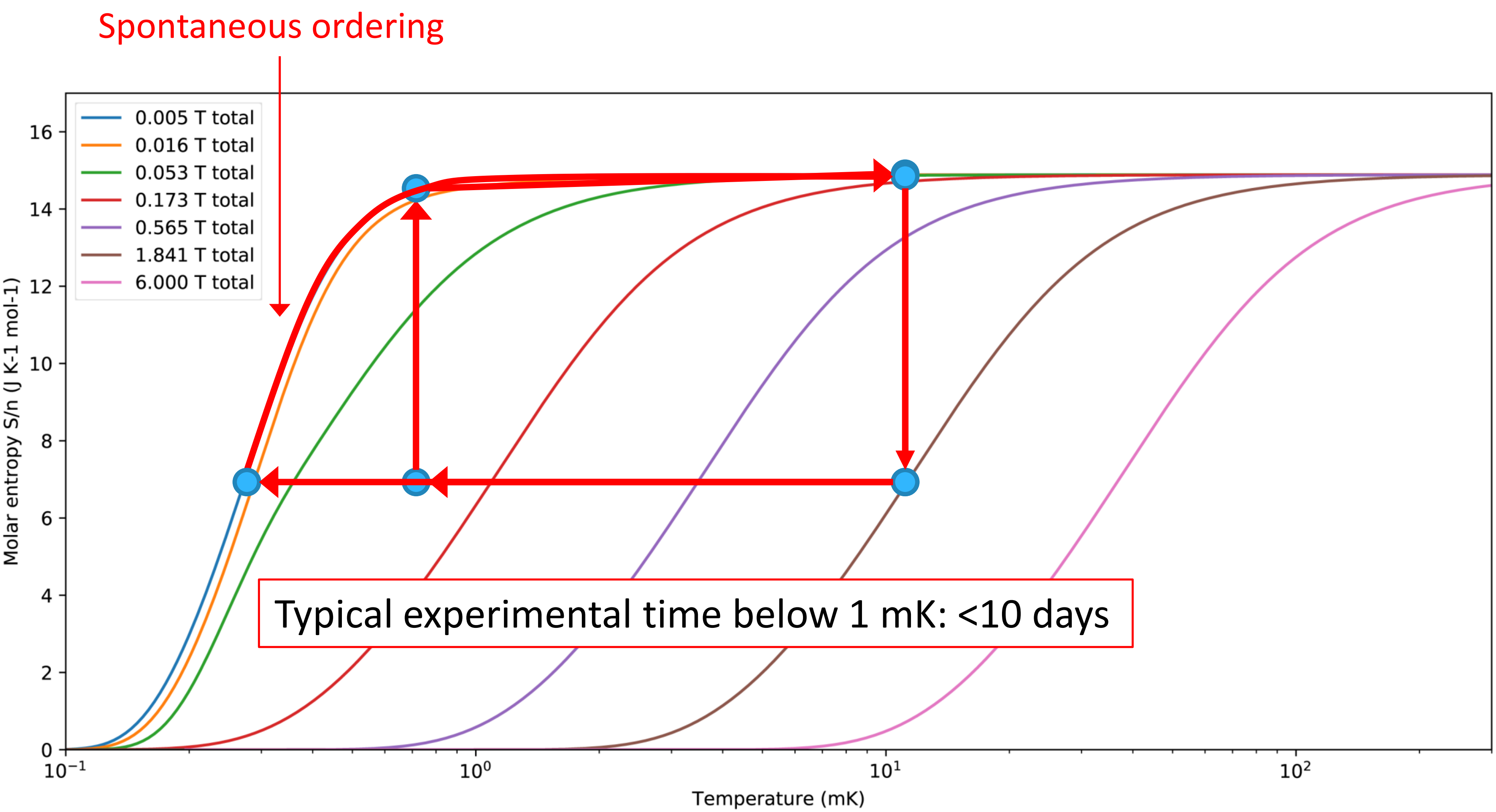
- Material properties at very low T (glasses)
- Nanodevices in quantum-mechanical ground state
- Superfluidity of ^{3}He, including topologically confined phases (microfluidic structures)
- Superfluidity in ^{3}He - ^{4}He mixtures?
- Space applications, bolometric detectors

Related work:

- CNDR: H. Fukuyama, Tokyo (…space applications)
- Continuous electronic demagnetization
- Cryogen-free refrigeration

Single-shot demagnetization: $PrNi_5$

Magnetic entropy vs. temperature in different B:



Thermalization times at low T

Homogeneous bulk elements:

$$\tau = \frac{\rho L^2 c(T)}{\kappa(T)} = \rho L^2 c(T) \rho_{th}(T)$$

For crystalline dielectrics at low T:

$c(T) \propto T$ electronic specific heat capacity

$\kappa(T) \propto T^3$ heat conduction by phonons

$\tau \propto T^{-2}$

e.g. silicon with L = 1 cm at T = 1 mK: $\tau \approx 2500$ s

Thermal boundary resistance (Kapitza resistance):

$$\tau = \rho L^2 c(T) \rho_{th}(T) + \rho V c(T) R_B(T) \qquad R_B(T) \propto T^{-n}; 3 \leq n \leq 5$$

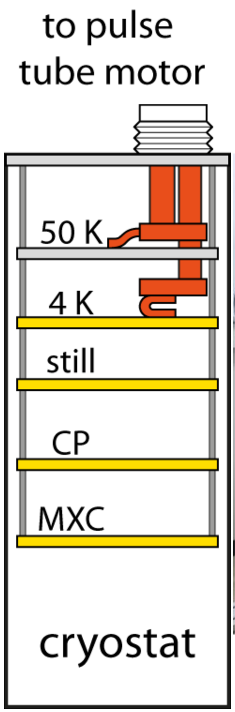
Thin elements (Si wafers, micro-/nano- machined devices):

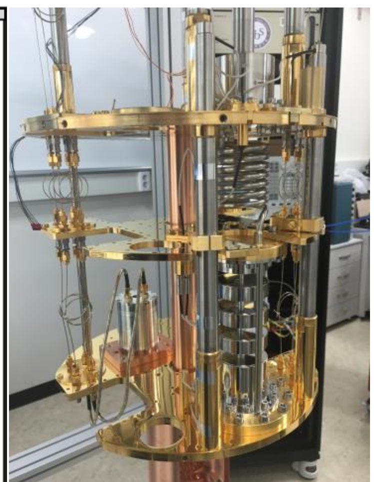
- additional scattering of phonons on surfaces
- reduced phonon mean free path -> suppressed thermal conductivity
- τ easily in excess of 10 days!

Continuous nuclear demagnetization refrigerator

(development at Institut Néel, Grenoble)

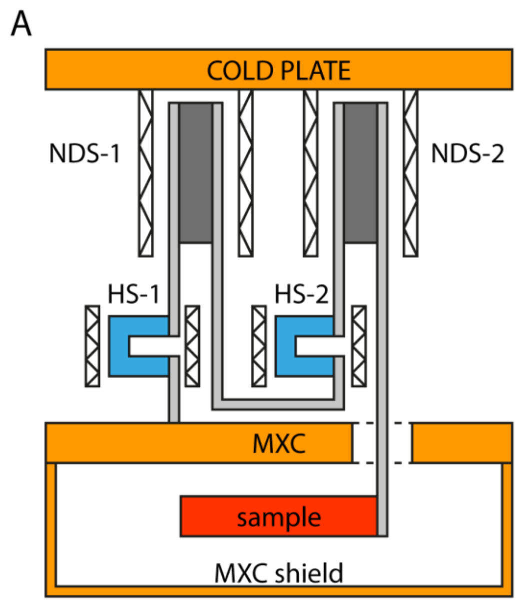
Dry dilution Refrigerator (DR):



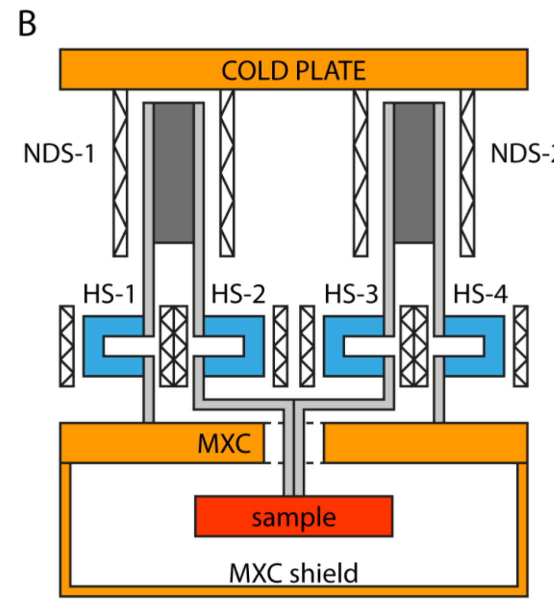


BlueFors LD-400

+ series CNDR:



+ parallel CNDR:



Goal: produce continuous cooling below 1 mK starting from ~10 mK at the MXC plate

Parts: 1 DR + 2 SC solenoids, 2 demagnetization stages, 2 or 4 heat switches, thermal links

Refrigerants: Cu, $PrNi_5$, Al

Estimating the CNDR performance

Single-shot nuclear demagnetization refrigerator (< 1 mK):

- analytical calculations usually suffice, taking into account removed entropy, losses and spin-electron interactions (Lounasmaa, Pobell)
- limiting factors = refrigerant & magnetic field, heat leaks (near equilibrium conditions)

Continuous adiabatic (electronic) demagnetization refrigerator (~100 mK):

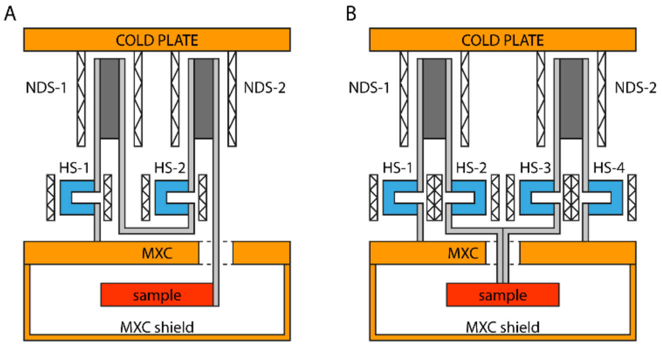
- analytical: good estimate of cooling power (entropy), optimum cycle duration (losses)
- series vs. parallel setup: starting and target temperature, cooling power, refrigerants
- usual limiting factors = as above + HEAT SWITCHES (off state)

CNDR (~1 mK):

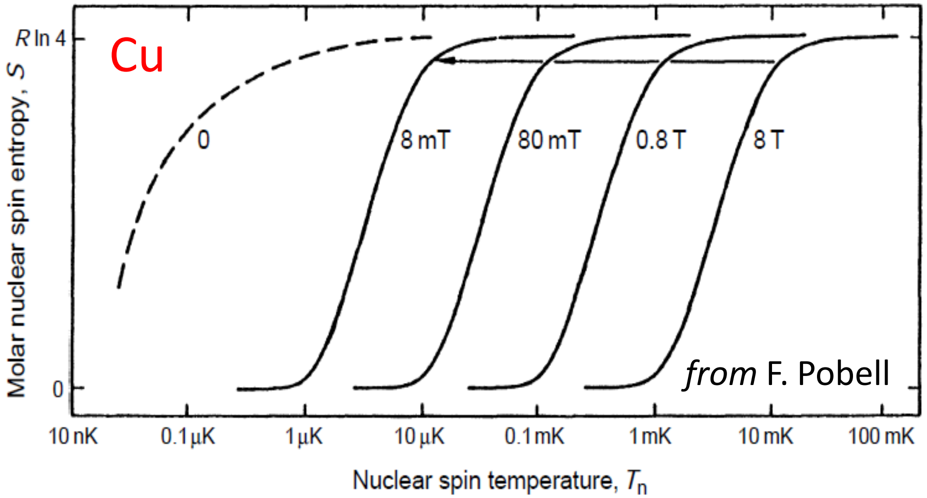
- needs to transfer significant heat between parts rapidly at very low temperature
- series vs. parallel setup: decision also affected by requirements on thermal resistances
- usual limiting factors = as above + HEAT SWITCHES (on state) + THERMAL LINKS

Optimization = tuning the dynamical interplay of demagnetization and heat transport processes

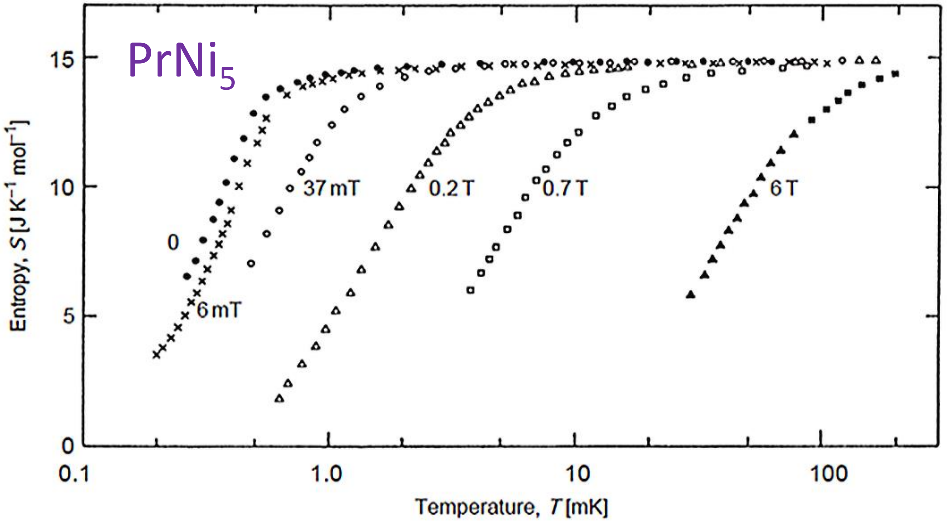
Time resolved numerical simulations!



Nuclear refrigerants



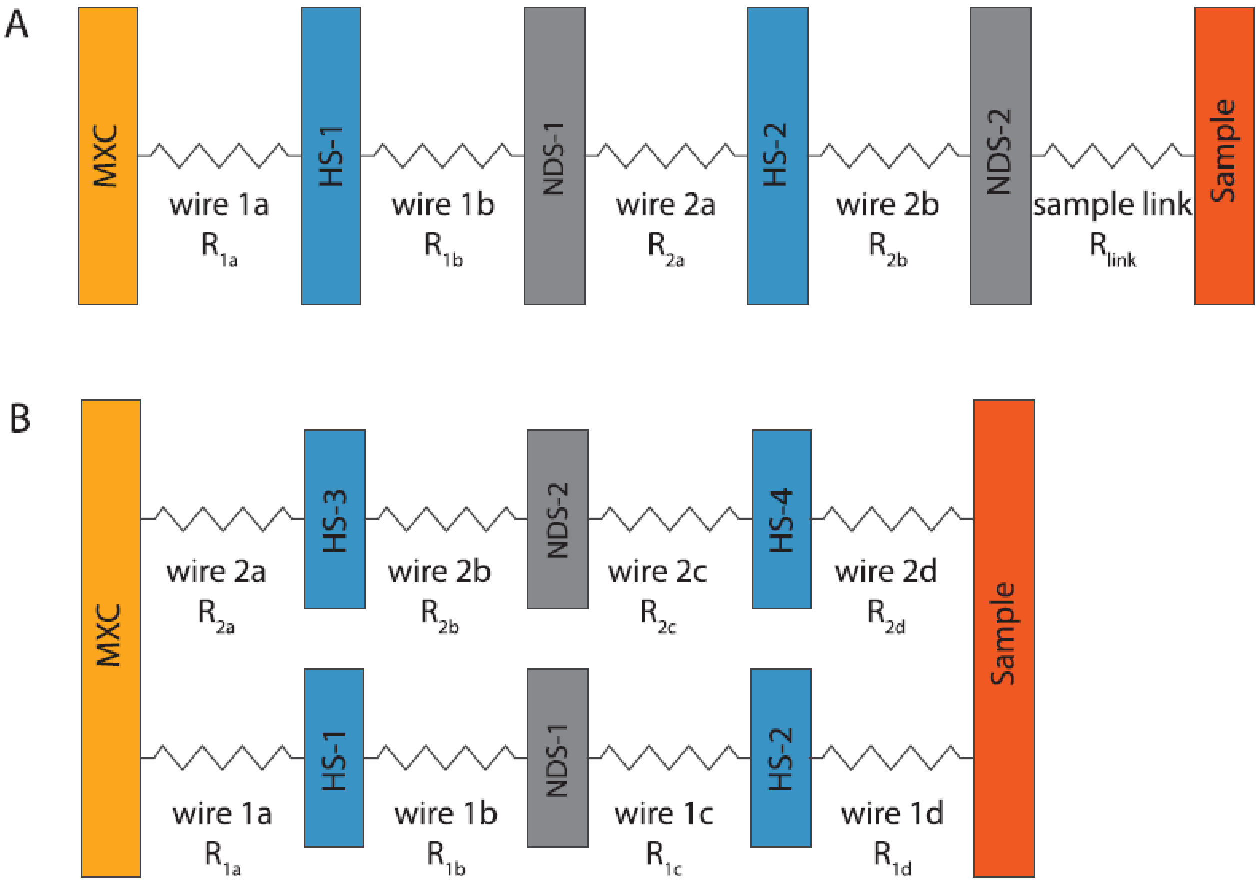
+ Low ordering temperature (< 1 μK)
+ Easy machining, contacting techniques
+ High purity (RRR up to 10000)
- High initial field required (~8 T)
- High τ_1, long thermal relaxation N-e
- Serial setup not feasible!
- Low demag rates (eddy currents)



+ Low initial field (compact magnets)
+ Higher demag rates (cooling power)
+ Hyperfine enhancement, fast relaxation
+ Works in serial setup
- Superconducting solder used (Cd, In,…)
- No machining, brittle
- Low purity (RRR ~ 30), magnetic phases

H.R. Folle, M. Kubota, Ch. Buchal, R.M. Mueller, F. Pobell, Phys. B Cond. Matt. 41, 223 (1980)

CNDR thermal models



Serial CNDR

Parallel CNDR

D. Schmoranzer, J. Butterworth, S. Triqueneaux, E. Collin, A. Fefferman, Cryogenics **110**, 103119 (2020)

Thermal conductances and heat leaks

Wires and thermal links (Cu, Ag):

- Temperature dependent thermal conductance modeled via an equivalent electrical resistance using Wiedemann-Franz law: $G = \frac{LT}{R}$, $L = 2.44 \times 10^{-8}$ W Ω K^{-2}
- Wire temperature = average of both ends

Superconducting heat switches (Al):

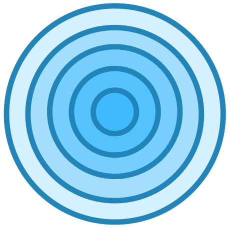
- Separate models for N and S states complying with $G_N \propto T$, $G_S \propto T^3$
- Values for aluminium heat switches adapted from Pobell

Demagnetization stages ($PrNi_5$):

- Model electronic and nuclear temperatures $T_e(t)$, $T_n(t)$ separately
- Uniform temperatures in the entire $PrNi_5$ rod *vs.* Finite heat conductivity
- Electron-nucleus heat exchange derived from Korringa law and heat capacities
- Heating sources: eddy currents $\propto \left(\frac{dB}{dt}\right)^2$ + vibrations $\propto |B|$ + constant heat leak

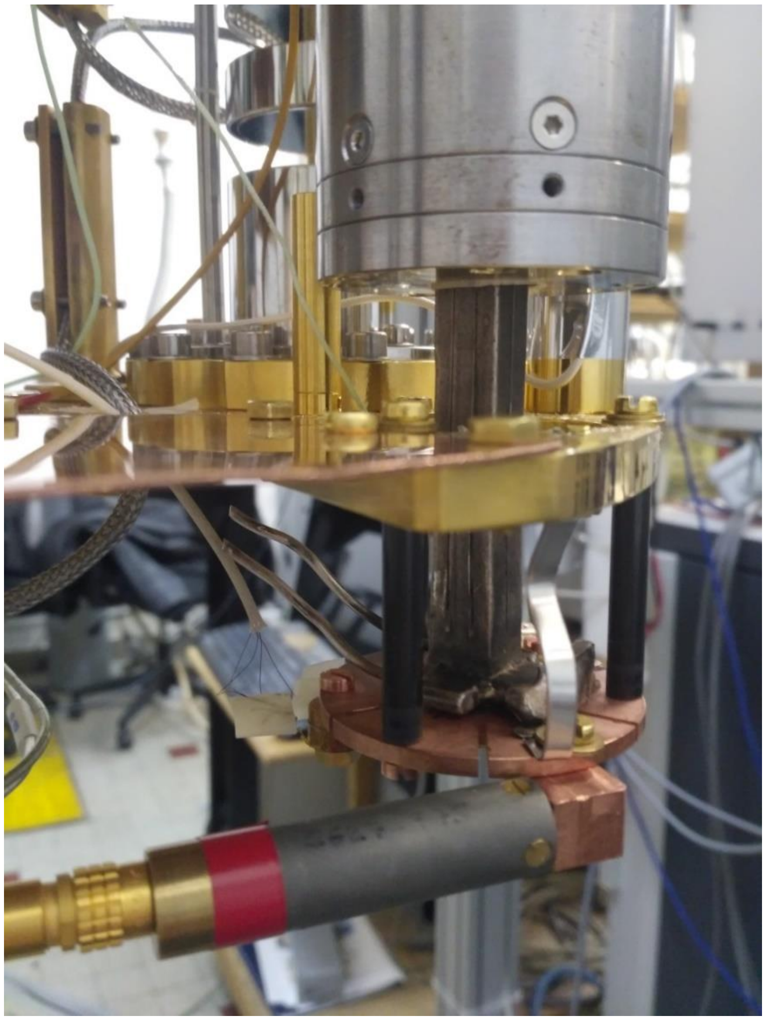
$PrNi_5$ rods: $T_e(r), T_n(r)$ dependence significant at time scales shorter than thermal relaxation time.

Layered demag stage model!



D. Schmoranzer, R. Gazizulin, S. Triqueneaux, E. Collin, A. Fefferman, JLTP **196**, 261 (2019)

Estimating parasitic heating



Testing setup with 0.1 mol $PrNi_5$

Heat leaks from literature:

- Constant heat leak (sample): 5, 10, or 20 nW
- Eddy current heating: $\dot{Q}_{ec} = 0.03\ [WT^{-2}s^2]\ \dot{B}^2$
- Vibrational heating: $\dot{Q}_v = 10^{-8}\ [WT^{-1}]\ |B|$
- Constant heat leak: 2 nW

J. M. Parpia, W. P. Kirk et al., Rev. Sci. Instr. **56**, 437 (1985)

Direct measurements:

- $PrNi_5$ stage Cd soldered to Ag rods
- Custom built compact 2.7 T solenoid
- SQUID noise thermometer MFFT-1

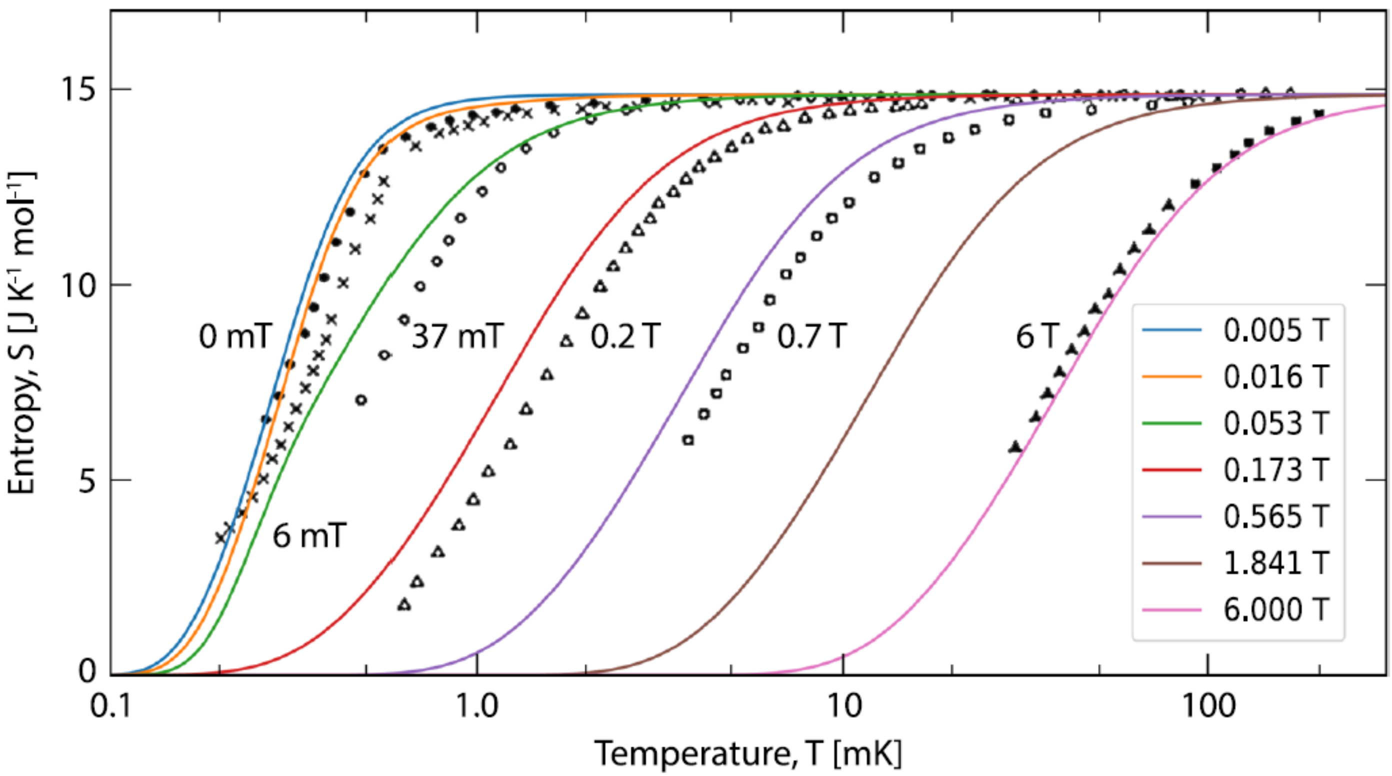
1st run: T_f = 2.2 mK
Heat conduction through thermometer cable!

Superconducting cable extension made.

2nd run: T_f < 1.1 mK
Thermometer at its noise sensitivity limit.

$PrNi_5$ model properties

Weakly interacting dipoles model is insufficient to match experimental values of entropy.
An empirical model was developed to approximate S(B,T); mismatch only at very low fields.



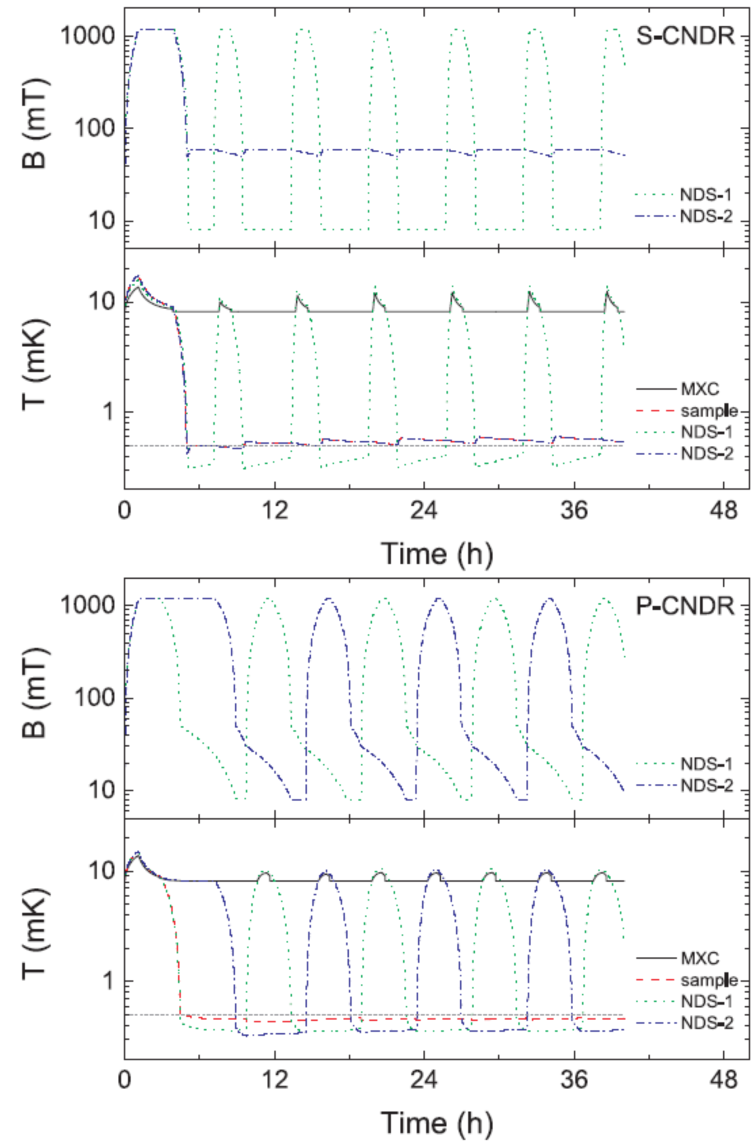
M. Kubota, H.R. Folle, C. Buchal, R.M. Mueller, F. Pobell, Phys. Rev. Lett. **45** (20), 1812 (1981)
D. Schmoranzer, R. Gazizulin, S. Triqueneaux, E. Collin, A. Fefferman, JLTP **196**, 261 (2019)

Heat conductivity from: H.C. Meijer, G.J.C. Bots, H. Postma, Physica 107B, 607–608 (1981).

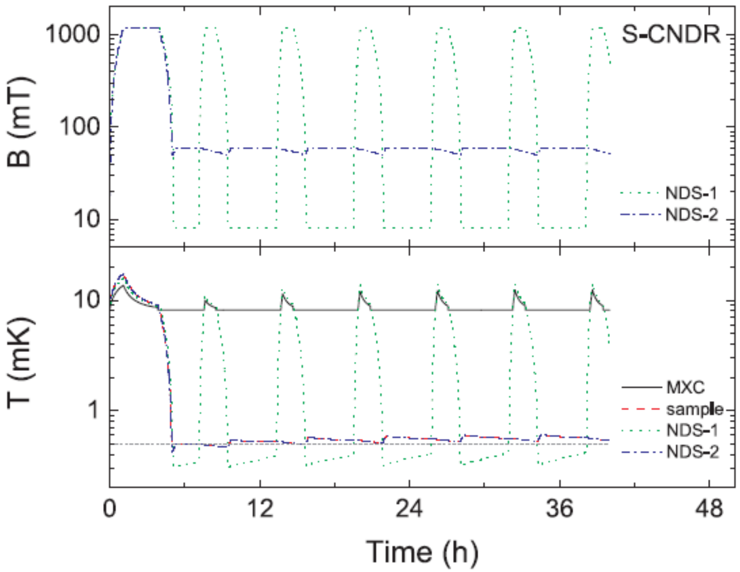
CNDR simulations

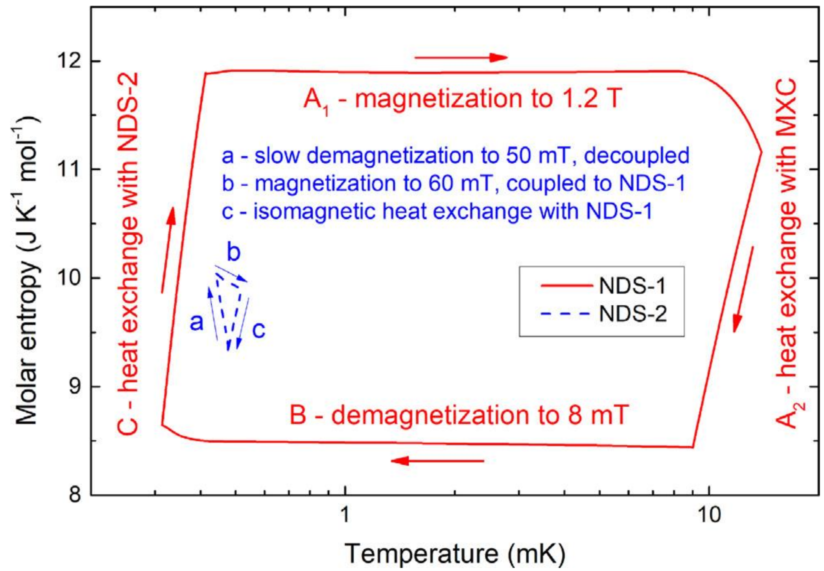
Implementation & procedure:

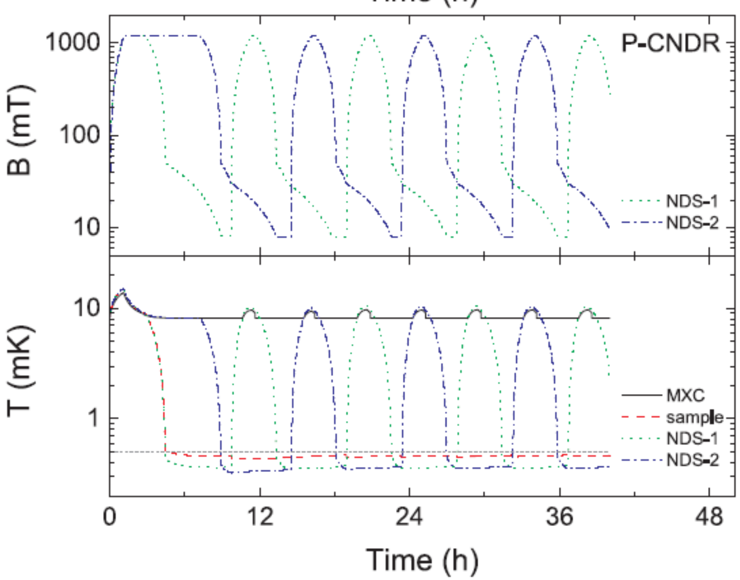
- Python: scipy.integrate.solve_ivp()
- Sample heat leak fixed at 5, 10, or 20 nW
- CNDR operation scripted in a custom built "language" using commands such as: "AD1 0.2T, 5000s", WAIT T1<Ts, HS1 ON…
- Linear, exponential and parabolic demag profiles implemented, B_{max} = 1.2 T
- Total cycle duration estimated based on heat leaks and removed entropy/cycle
- Durations of individual steps tuned by hand
- Time-traces of all temperatures obtained, additionally monitoring heating and entropies
- Stabilization after several cycles

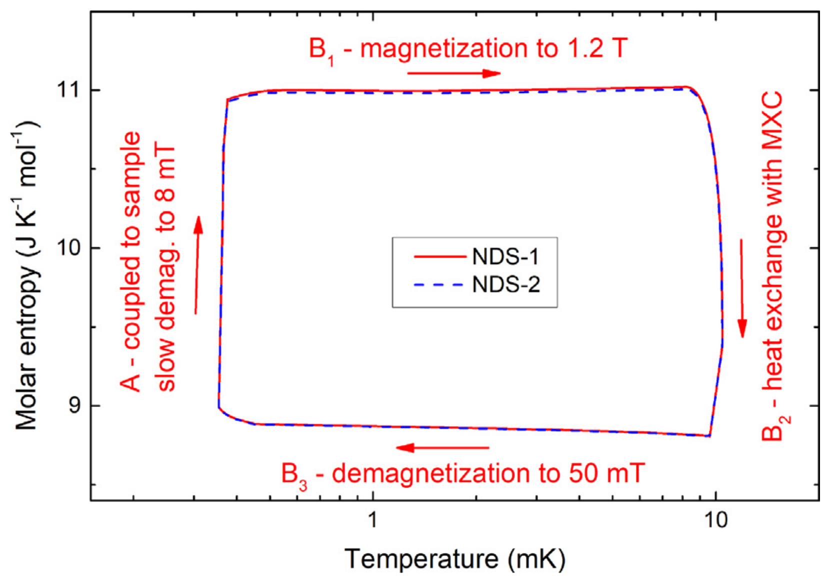


Parallel and serial CNDR, 5 nW load

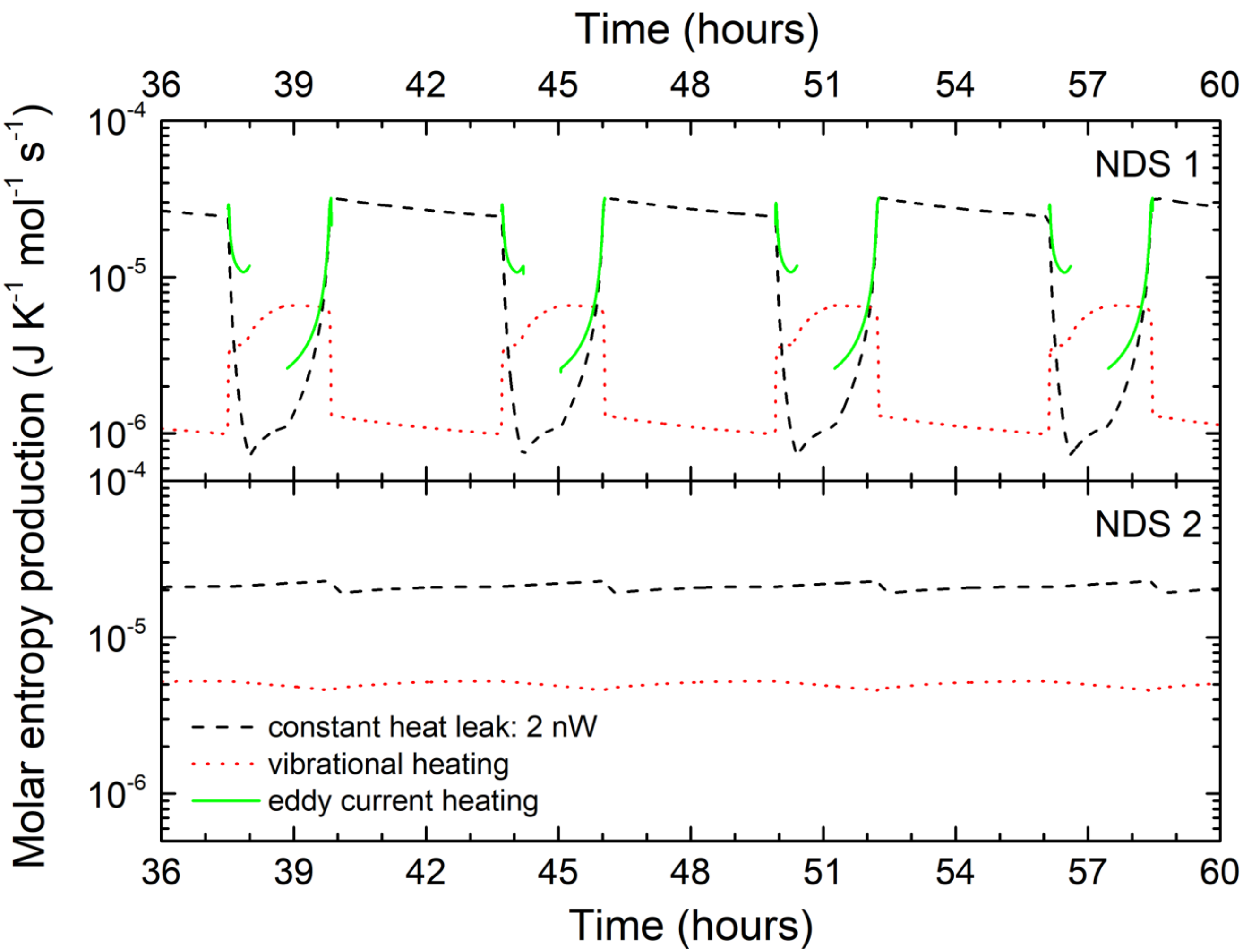




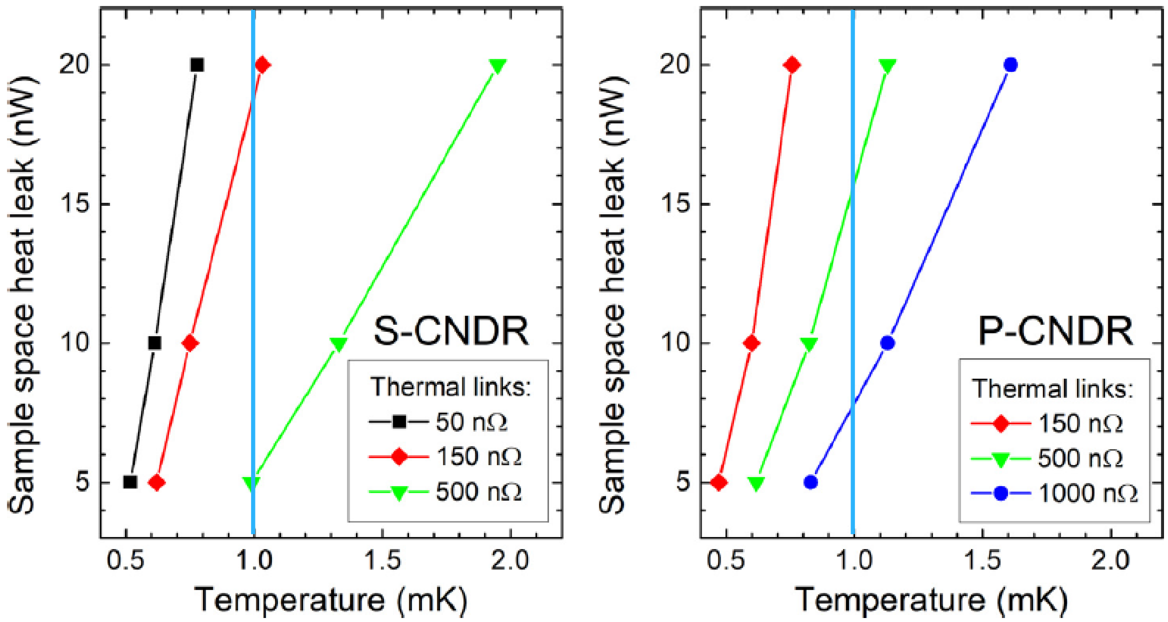




Serial CNDR, entropy production rates



Parallel and serial CNDR performance



Criteria:

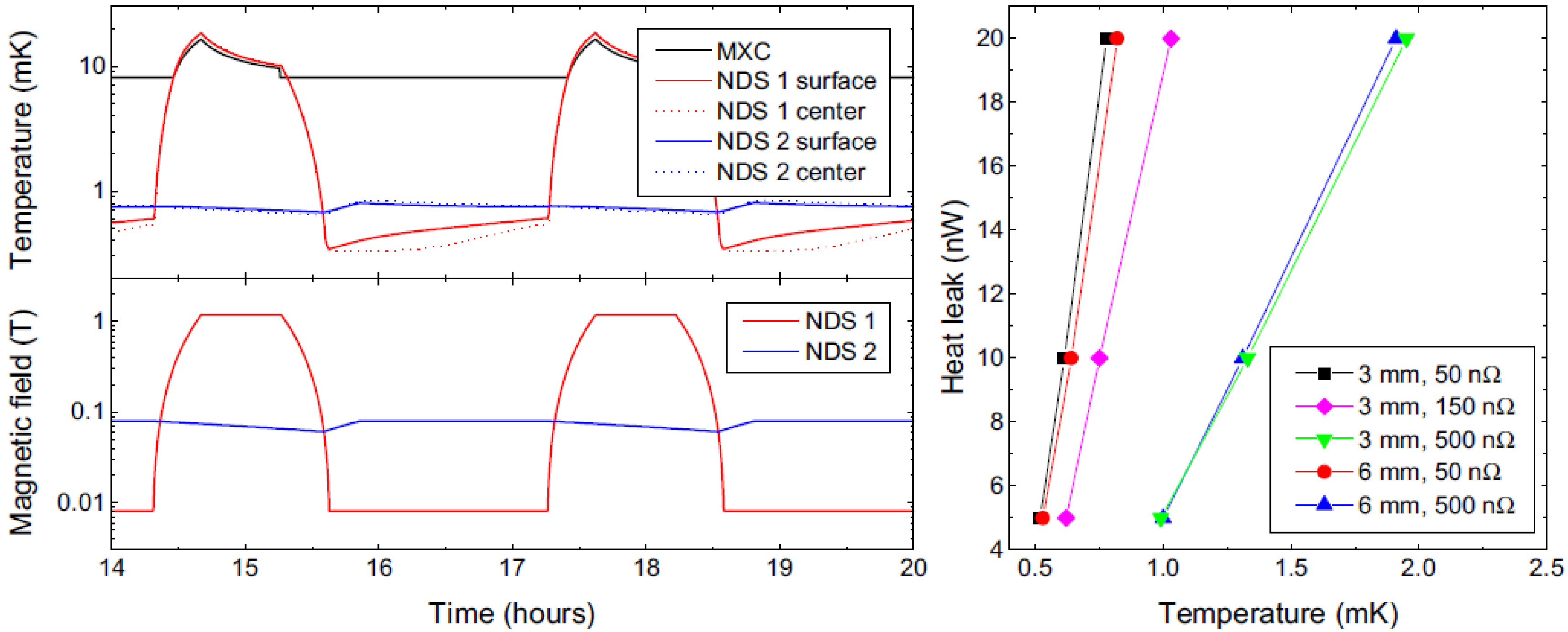
Sample temperature vs. sample heat load and thermal link resistance

Outcome:

- Sub-mK temperatures achievable in both serial and parallel setups
- Thermal link resistance determines the available cooling power
- Especially the serial CNDR setup requires very high conductivity links (precooling a pool of high-capacity nuclei in the second stage from the first stage)

D. Schmoranzer, J. Butterworth, S. Triqueneaux, E. Collin, A. Fefferman, Cryogenics **110**, 103119 (2020)

Size of $PrNi_5$ rods, layered stage model



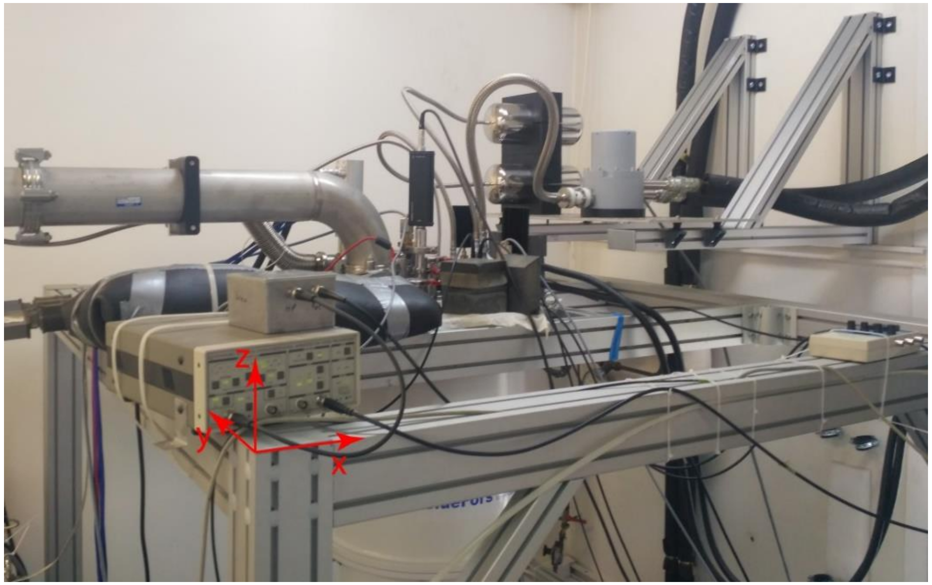
Outcome:

- Reducing the size of the “standard” 6 mm rods: weak effect, largely irrelevant (except for boundary thermal resistance)
- Observable difference between center and surface electronic temperature
- Relevant thermalization time scales: 1-2 hours (depending on temperature)

Vibrations & Accelerometry

Setup:

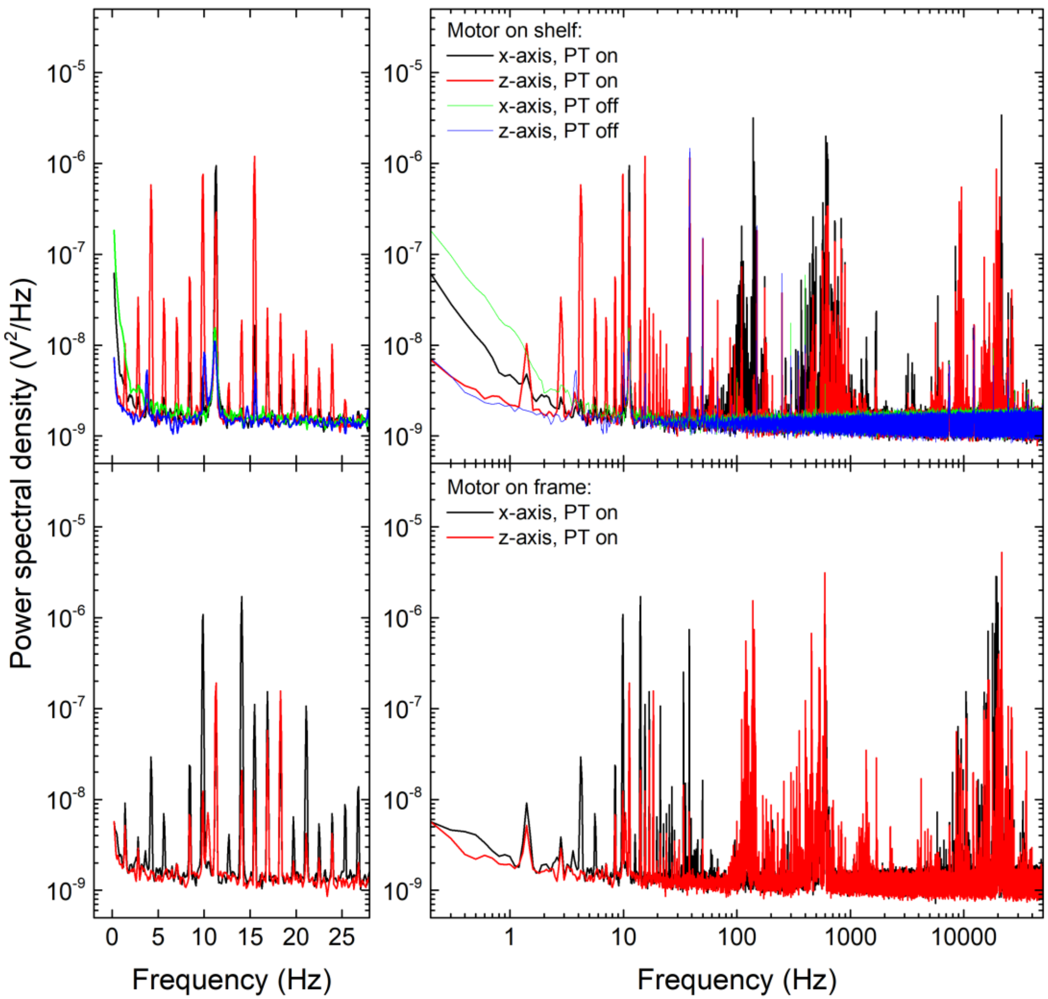
- Dilution fridge: BlueFors LD-400
- Accelerometers: 3-axis (P.-E. Roche)
- PCB 351B41 (1-2000 Hz) : x, z- axes
- Kistler 8703A50M8 (1-5000 Hz): y-axis
- Amplifier: PCB 480B21 (gain 10)



Measurements:

- Taken at: Top flange / motor plate / MXC
- Effect of adding sandbags / lead bricks, frame air legs either lowered / raised
- Motor placed on the frame vs. on a separate shelf
- Solid pipes vs. flexible tubes to compressor
- Final results: average of 100 individual spectra, signals recorded at 100 kS/s (10 s each)

Accelerometry – MXC

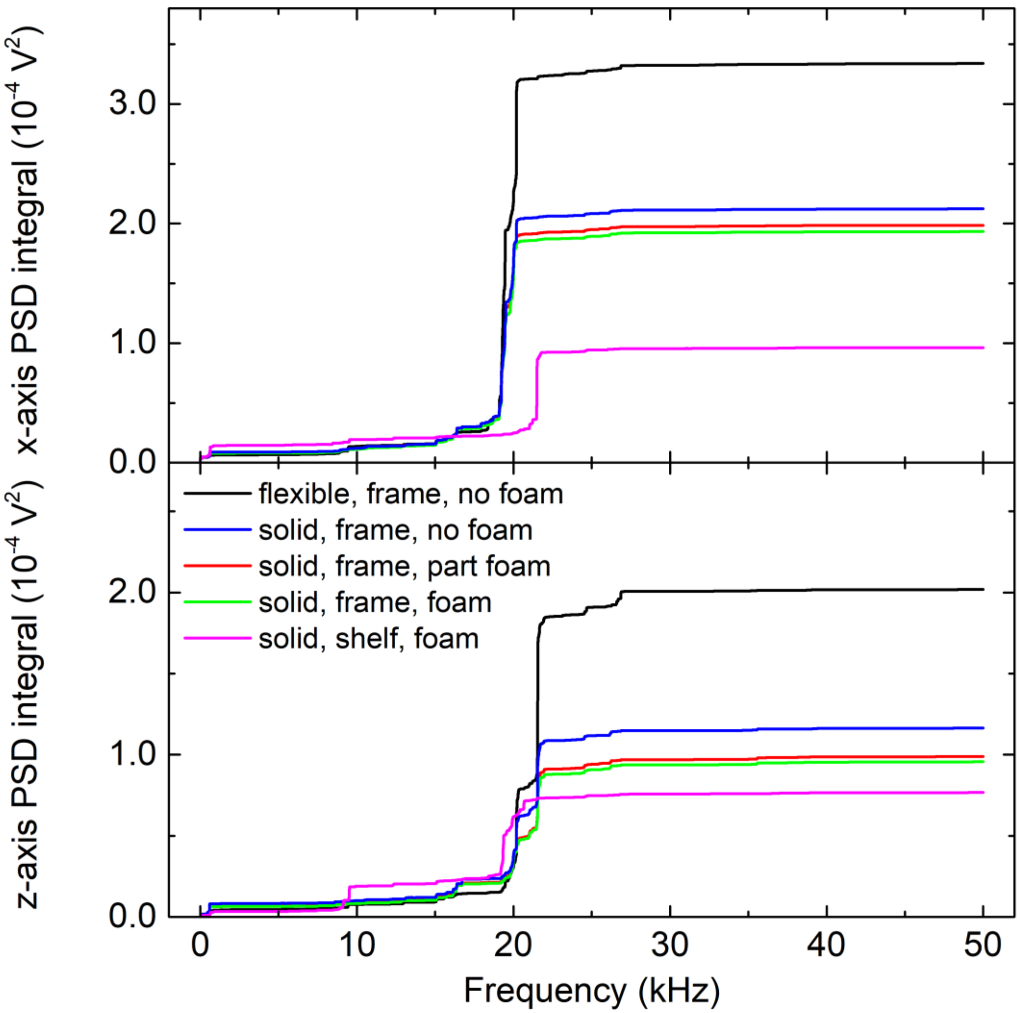


Results:
With PT off, the vibrations disappear. The massive signal at 20 kHz is not an artifact and is caused by the pulse tube.

Notes:
Due to lack of calibration above 2 kHz, only relative comparison is possible. May be affected by sensor resonances.

D. Schmoranzer, A. Luck, E. Collin, A. Fefferman, Cryogenics **98**, 102–106 (2019).

Accelerometry – MXC



Conclusions:

- Ideal configuration: Solid pipes with foam isolation, motor on a separate shelf.
- Focus on high-frequency acoustic noise ~20 kHz
- Measurements with high-frequency accelerometers

D. Schmoranzer, A. Luck, E. Collin, A. Fefferman, Cryogenics **98**, 102–106 (2019).

Conclusions

- Numerical simulations complete, describe the system adequately
- $PrNi_5$: both parallel and serial setups possible
- Practical tests with $PrNi_5$ under way
- Thermal links and heat switches – talk by Sébastien Triqueneaux
- To resolve: vibrational heating (indications from accelerometry)