

# Chapter 4

## Scanning Tunneling Spectroscopy

Objective: to learn the implementation principles of STM spectroscopy and some of its main applications

# The spectroscopy mode

Measuring the differential conductance gives access to the LDOS.

$$\frac{dI}{dV} \propto \int_{-\infty}^{+\infty} N_B(E) \frac{df(E - eV)}{dV} dE$$

$$\text{If } T = 0: \frac{dI}{dV}(V) \propto N_B(E_F - eV)$$

A perfect stability is required:

If the tip-sample distance varies by  $\Delta z$ , then the current  $I$  varies by  $\Delta I$ :

$$\frac{\Delta I}{I} = 2\alpha\Delta z = 2\frac{\sqrt{2mW}}{\hbar}\Delta z$$

with  $\alpha \approx 1\text{\AA}^{-1}$

To avoid this mechanical noise down to below 1% of relative variation  $\Delta I/I$  requires  $\Delta z$  below 0.5 pm, i.e. a pm-scale mechanical stability.



# The spectroscopy mode

How to make a spectroscopy experiment ? First:

- Position the tip
- Freeze height regulation

Second:

- Measure  $I(V)$
- Differentiate to get  $dI/dV$ , which measures the sample LDOS

*OR*

- Add a bias modulation  $dV$ .
- Measure ac current response amplitude as a function of  $V_{dc}$ .

$$V = V_{dc} + V_{ac0} \cos \omega t$$

$$I = I(V_{dc}) + \left. \frac{dI}{dV} \right|_{V=V_{dc}} \cdot V_{ac0} \cos \omega t$$

# Chapter 4

## Scanning Tunneling Spectroscopy

### 4.1 Superconductors

# Elements of superconductivity (1)

A superconductor (below  $T_c$ ,  $I_c$  and  $B_c$ ):  
zero resistance,  
perfect diamagnetism ( $B = 0$ ),  
modified density of states at the FL,

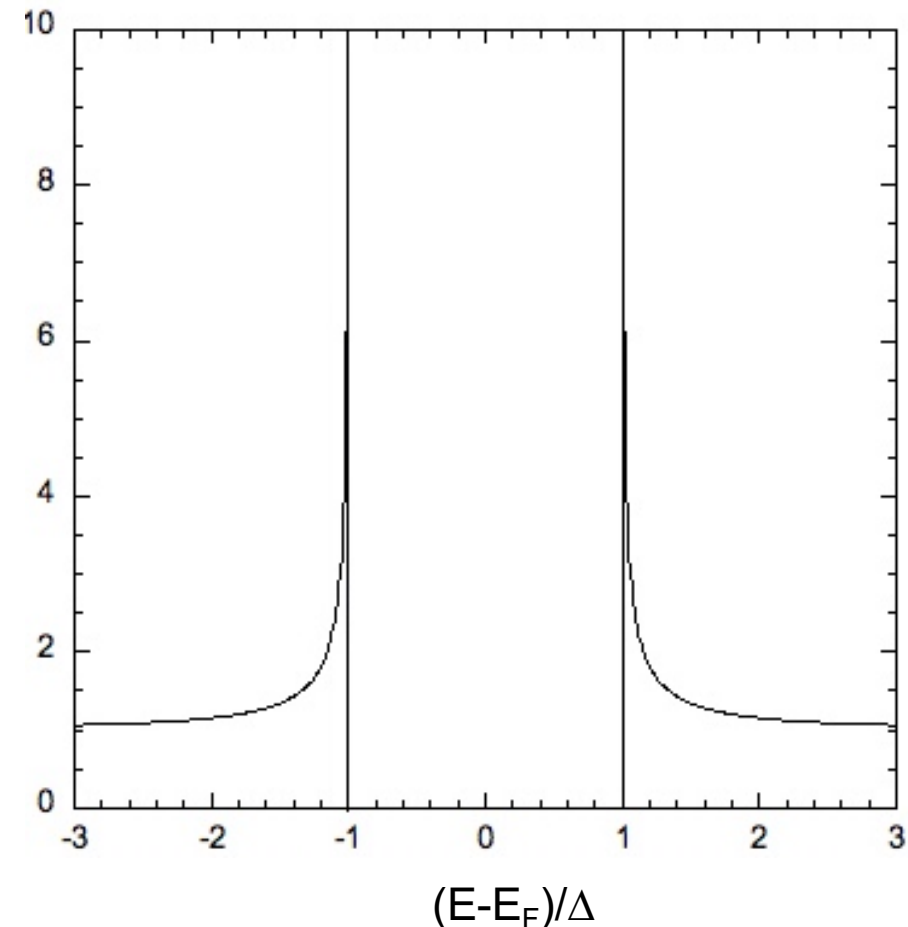
BCS theory : the DOS writes : DOS  
(a.u.)

$$N_S(E) = 0 \quad \text{if } |E - E_F| < \Delta$$

$$N_S(E) = N_N \frac{|E|}{\sqrt{(E - E_F)^2 - \Delta^2}} \quad \text{if } |E - E_F| > \Delta$$

The energy gap is :

$$\Delta(T \rightarrow 0) = 1.76 k_B T_c$$



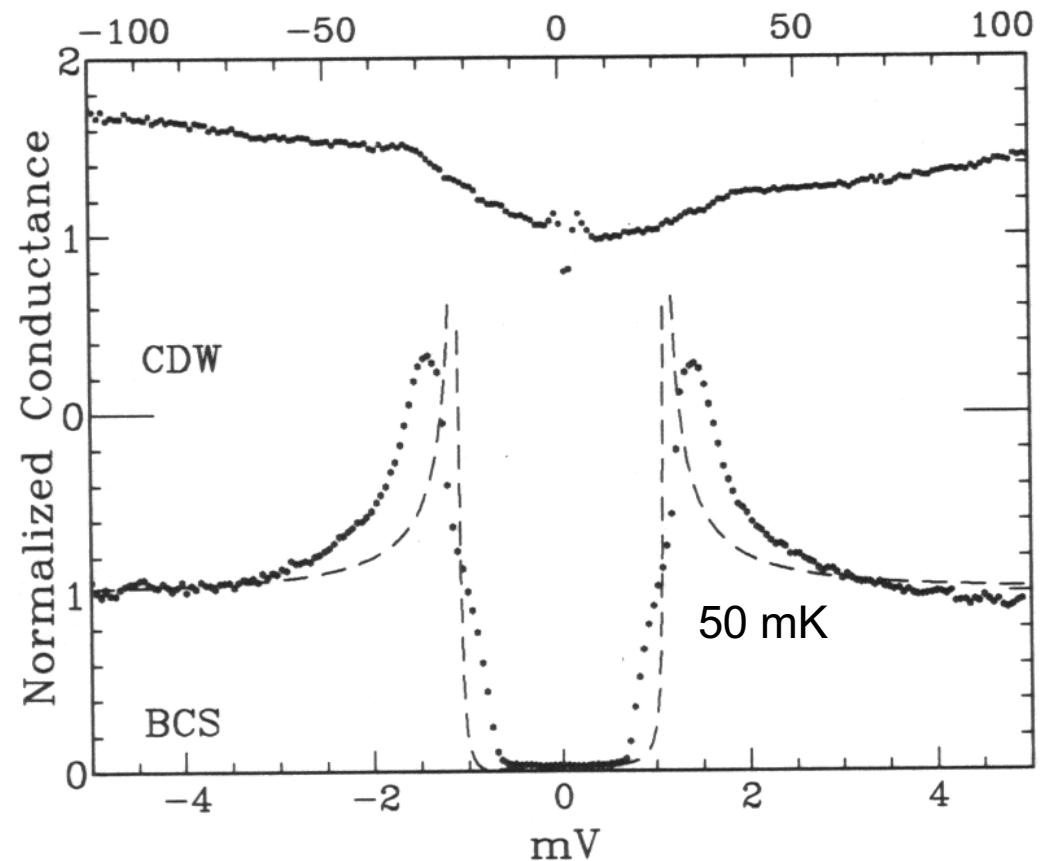
# NbSe<sub>2</sub>

Lamellar compound,

Easy to cleave (scotch tape): gives a clean, inert surface.

Superconducting below 7.2 K.

Spectra follows BCS shape.



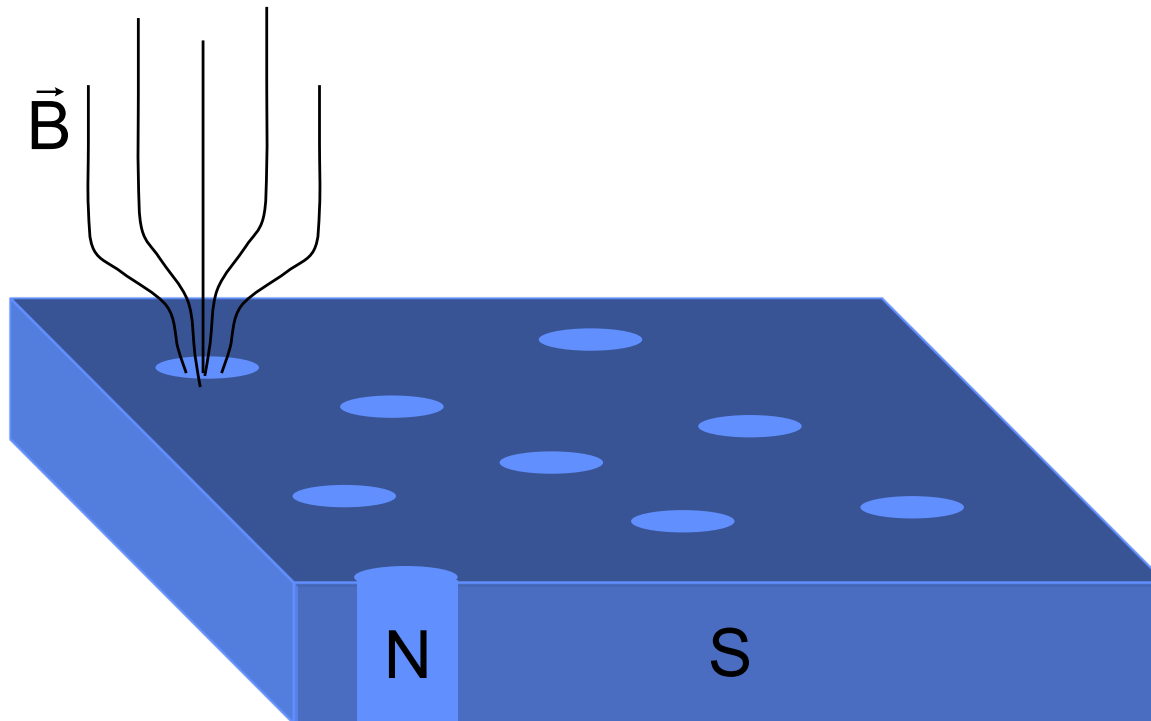
H.F. Hess et al., Physica B 169, 422 (1991).

# Elements of superconductivity (2)

A superconductor under magnetic field: magnetic field penetrates as vortices made of a normal core, each vortex carries a flux quantum  $\phi_0$ .

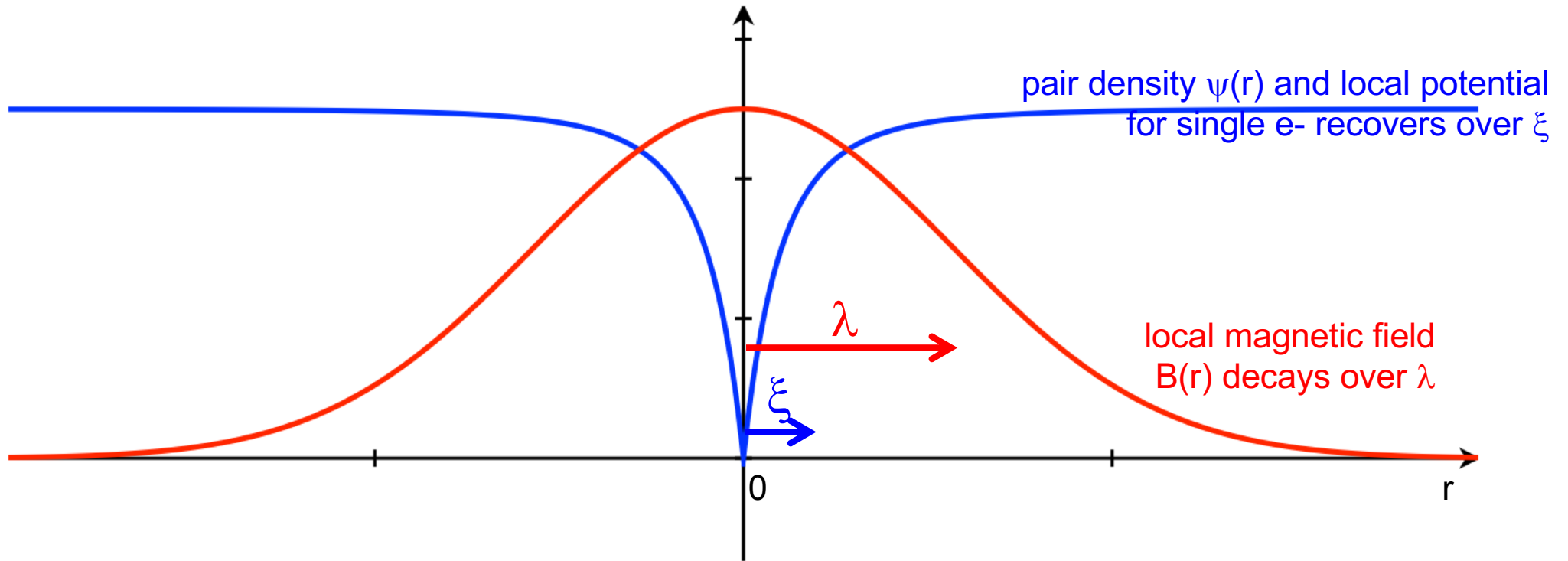
$$\phi_0 = \frac{h}{2e} = 2.0 \cdot 10^{-15} \text{ T.m}^2 = 2.0 \text{ mT.}\mu\text{m}^2$$

When field increases, the number of vortices increases accordingly.



# Inside a vortex

Local magnetic field depletes Cooper pair density and pairing potential.



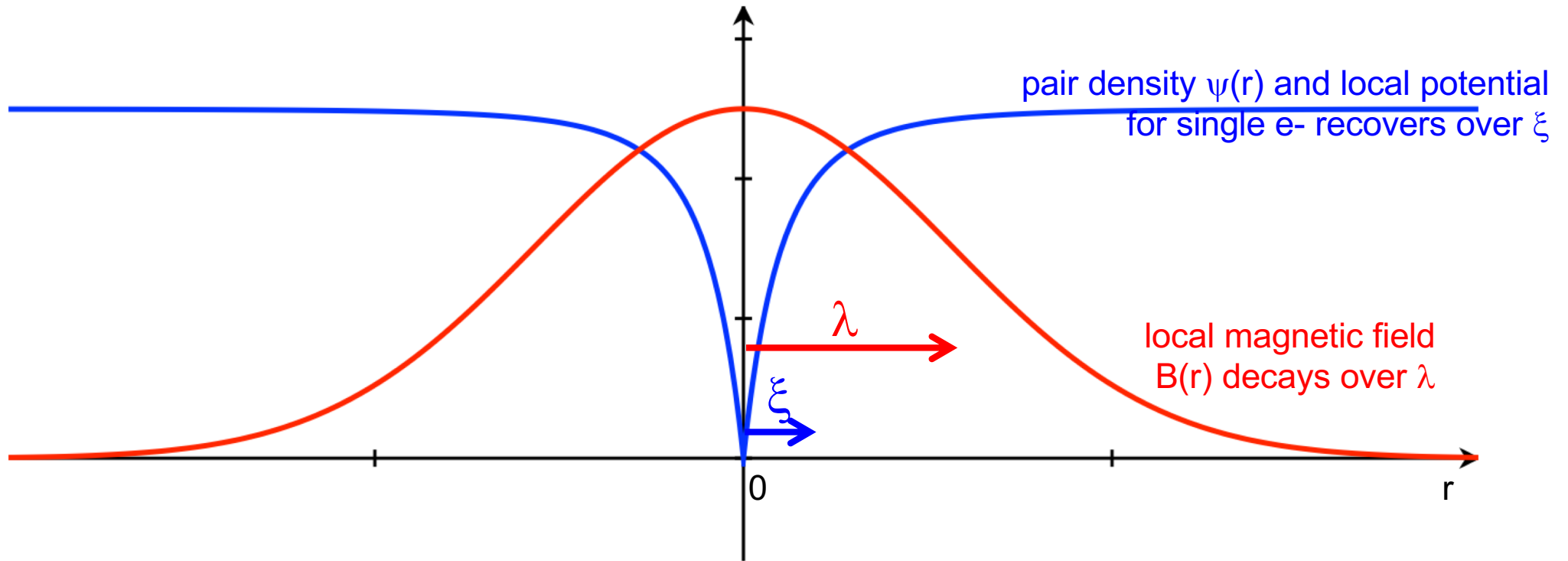
Magnetic field decays on the penetration length  $\lambda_L$ ,

Pair density decays on the coherence length  $\xi_s$ .

In type II superconductor:  $\lambda_L > \xi_s$  : a vortex is energetically favored compared to full screening. In NbSe<sub>2</sub>, :  $\xi_s = 77 \text{ \AA}$ ,  $\lambda_L = 2000 \text{ \AA}$ .

# Inside a vortex

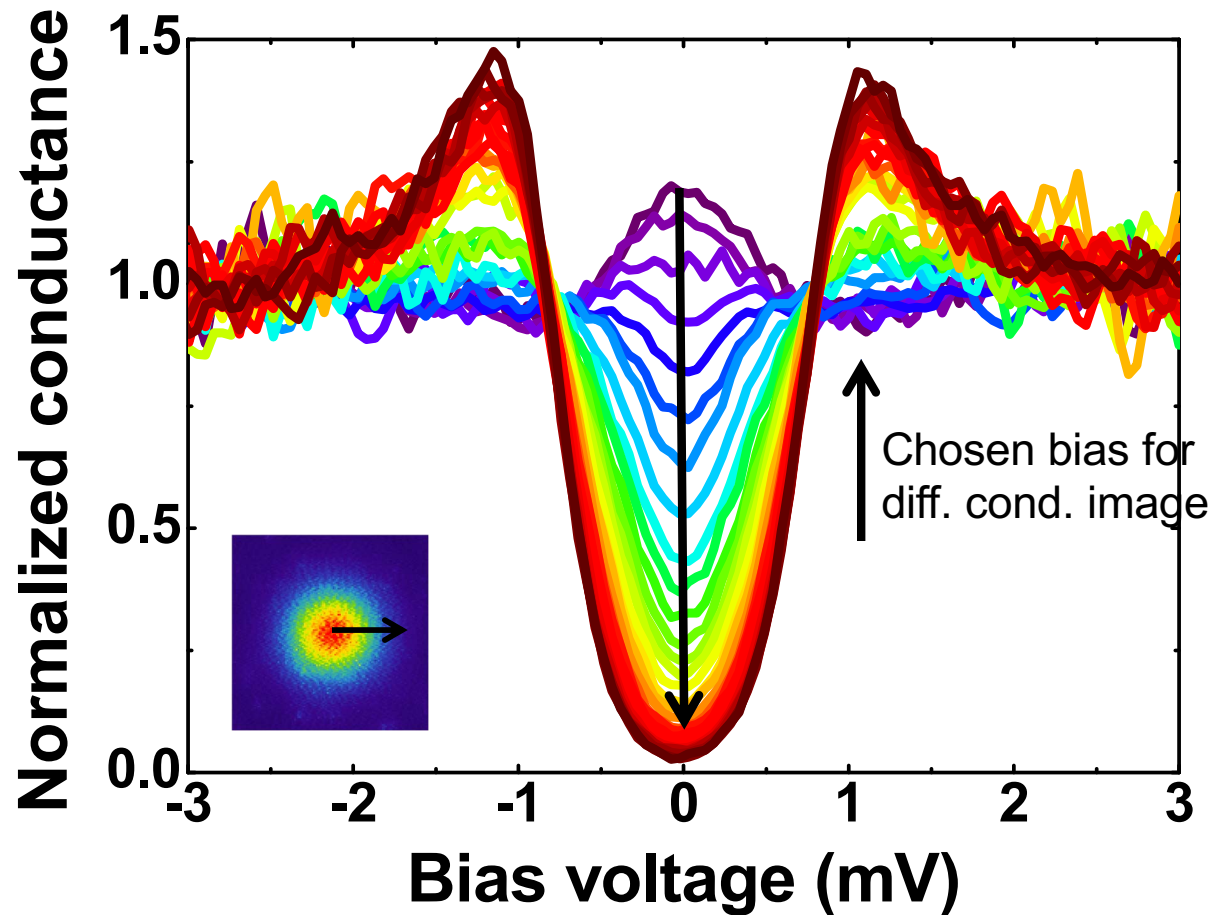
Local magnetic field depletes Cooper pair density and pairing potential.



Screening currents around a single point: a vortex.

Localized (single-)electron states, local density of states modified.

# Localized states in a vortex



Series of LDOS spectroscopy along a line across a vortex:  
quasiparticles states confined within a vortex core = LDOS peak.

A. Fente, H. Suderow et al., Phys. Rev. B **94**, 014517 (2016).



# The differential conductance imaging

During scan: ac modulation  $V_{ac}$  added to dc bias  $V_{dc}$ .

$$V = V_{dc} + V_{ac0} \cos \omega t$$

Regulation slower than ac modulation  ac current modulation.

$$I = I(V_{dc}) + I_{ac} \cos \omega t = I(V_{dc}) + \left. \frac{dI}{dV} \right|_{V=V_{dc}} \cdot V_{ac0} \cos \omega t$$

(first order expansion of  $I(V)$ ).

$dI$  is measured and displayed (with grey levels).

Bias  $V_{dc}$  well chosen so that  $dI/dV$  image reflects the LDOS structure.

Also called STS map.

# The vortex lattice

Abrikosov (Nobel 2003) vortex lattice.

Triangular geometry: interaction between vortices is minimized.

Vortex density determined by the magnetic field.

H.F. Hess et al., Phys. Rev. Lett. 62, 214 (1989).

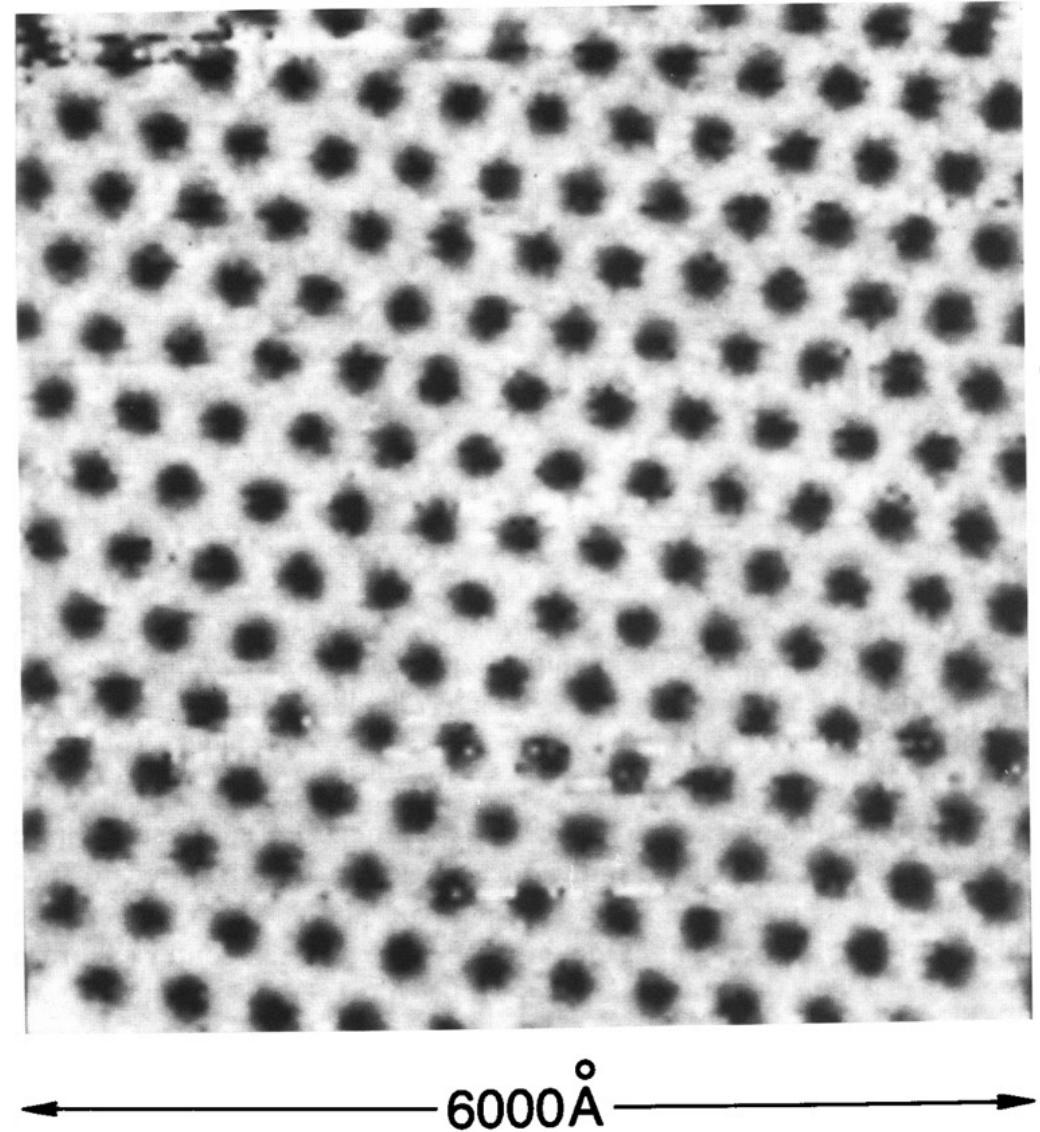


Fig. 4.120. Abrikosov flux lattice produced by 1 T magnetic field in NbSe<sub>2</sub> at 1.8 K. The gray scale corresponds to  $dI/dU$  (Hess *et al.*, 1989).

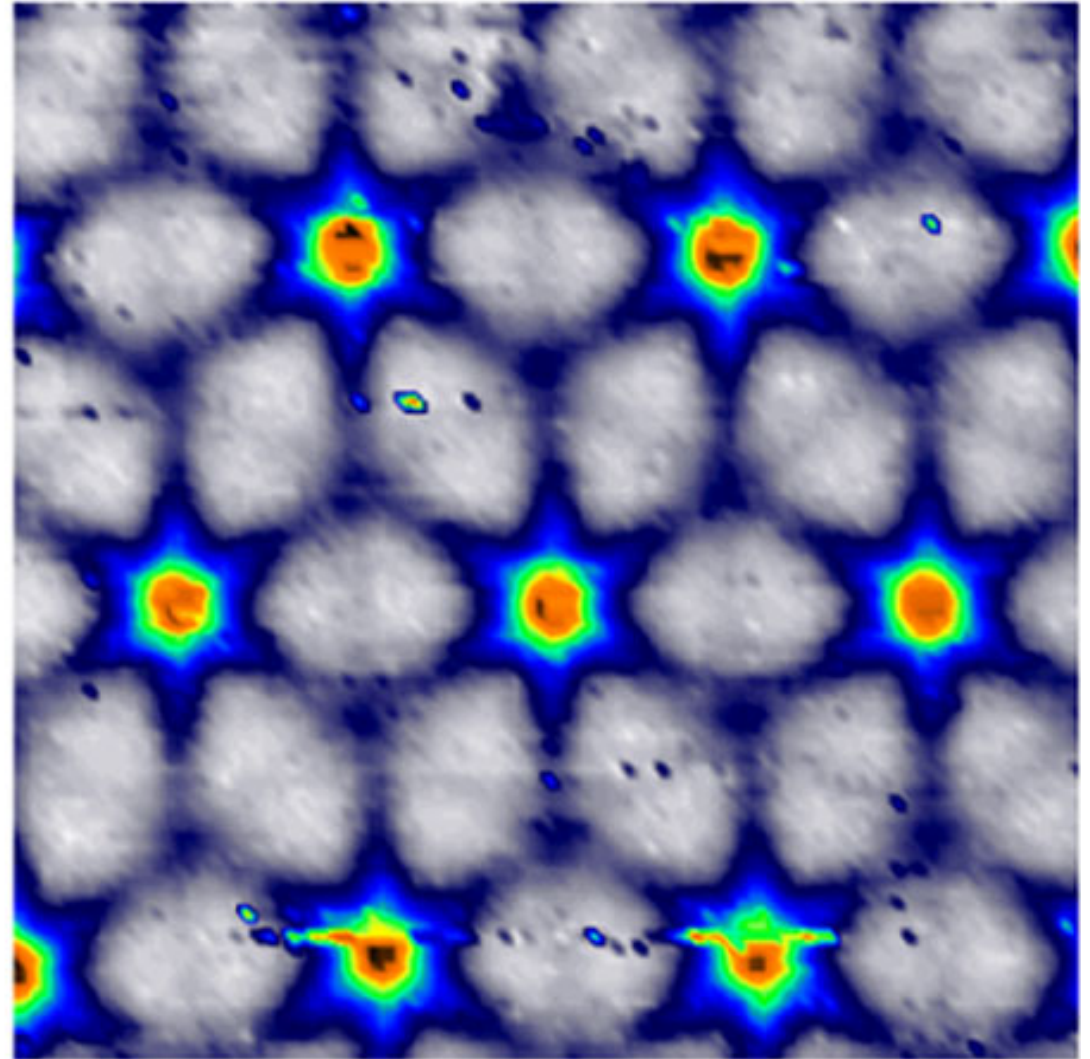
# The vortex lattice

Abrikosov (Nobel 2003) vortex lattice.

Triangular geometry: interaction between vortices is minimized.

Vortex density determined by the magnetic field.

I. Guillamon, H. Suderow et al., Phys. Rev. Lett. 101, 166407 (2008).



360 x 360 nm<sup>2</sup>, 0.1 K, 0.15 T

# High-T<sub>c</sub> superconductors

Ceramics made of CuO<sub>2</sub> planes and rare earth elements.  
d-wave condensate, strong disorder.

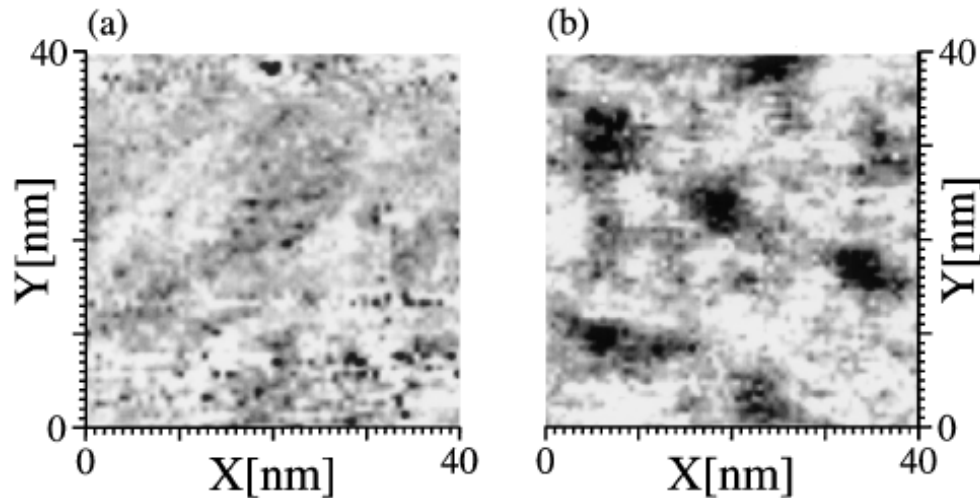


FIG. 1. STS maps of the conductance at  $V = -\Delta_p/e$  at 4.2 K on overdoped BSCCO ( $T_c = 74.3$  K); white and dark correspond to large and low conductance, respectively: (a)  $B = 0$  T; (b)  $B = 6$  T.

= diff. cond. map

I. Maggio-Aprile et al, Phys. Rev. Lett. 80, 3606 (1998).

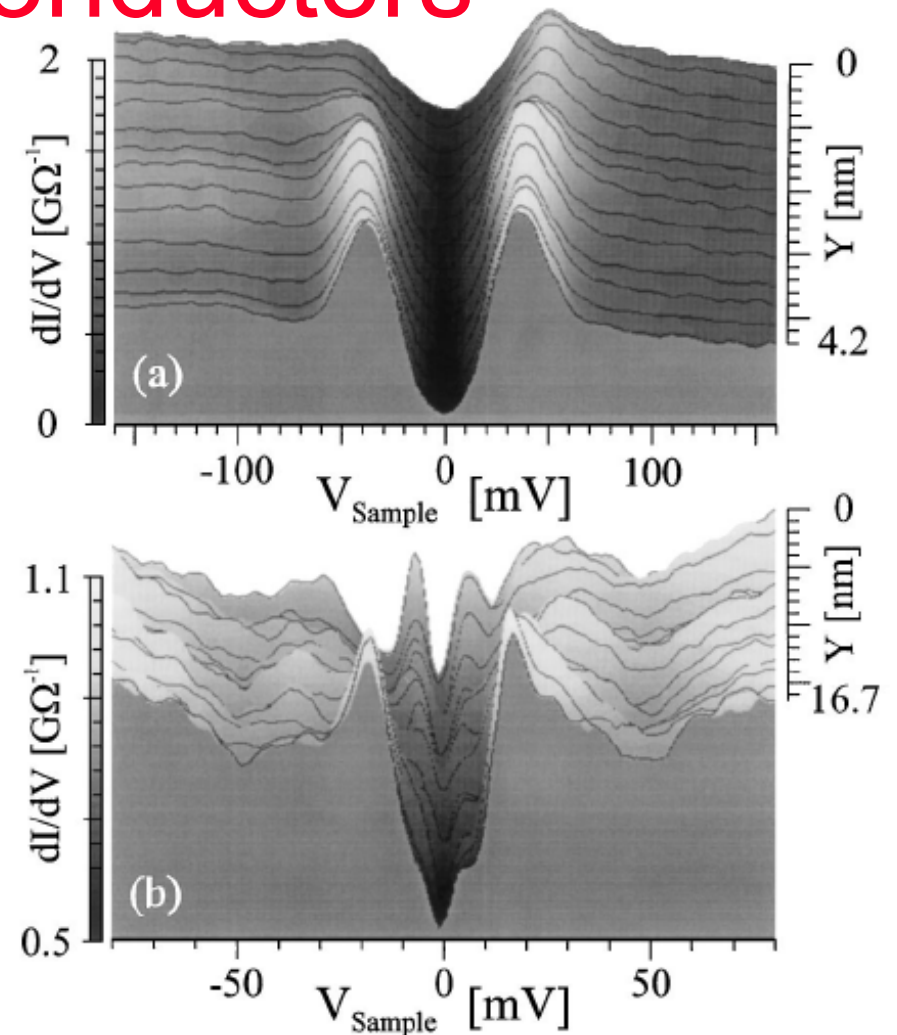


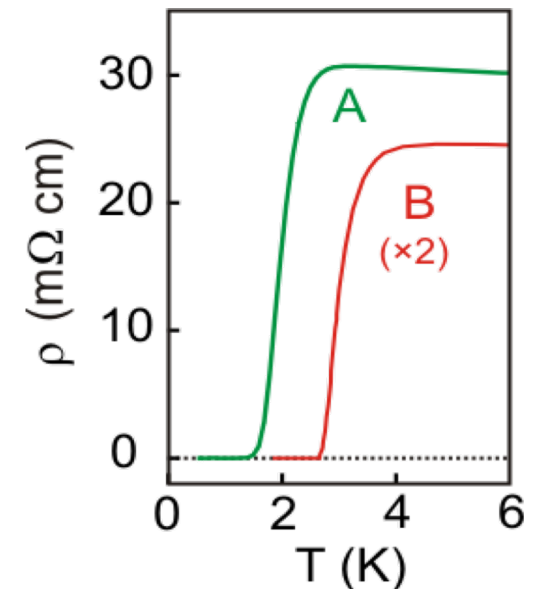
FIG. 2. Three dimensional view of tunneling spectra measured at equally spaced positions along a line extending from the center of a core ( $Y = 0$ ) to a point located away from the vortices at 4.2 K and 6 T. (a) 4.2 nm trace on overdoped BSCCO ( $T_c = 74.3$  K). (b) 16.7 nm trace on optimally doped YBCO ( $T_c = 91$  K). Note the difference by a factor of 2 in the energy scales and the large zero bias conductance in YBCO.



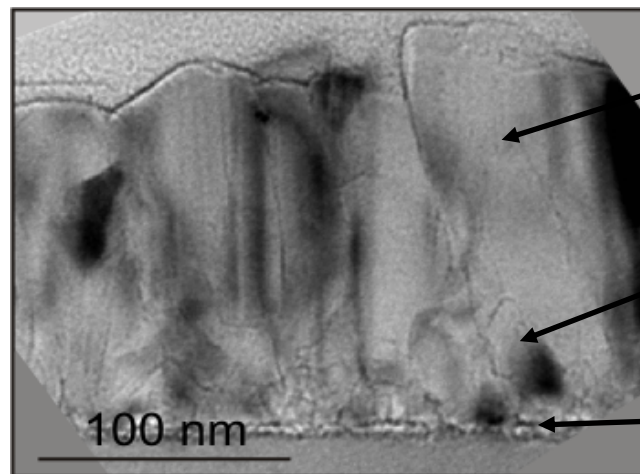
# Superconducting polycrystalline Boron-doped Diamond (poly-BDD)

Prepared by Microwave Plasma Enhanced Chemical Vapor Deposition: MWPECVD.

Sample	Substrate	Thickness	Resistivity	$T_C$
A	Quartz	1 $\mu\text{m}$	26 $\text{m}\Omega \cdot \text{cm}$	2 K
B	$\text{SiO}_2/\text{Si}$	200 nm	12 $\text{m}\Omega \cdot \text{cm}$	3 K



TEM cross-section image



Large columnar grains in (111) and (001) crystallographic orientations,

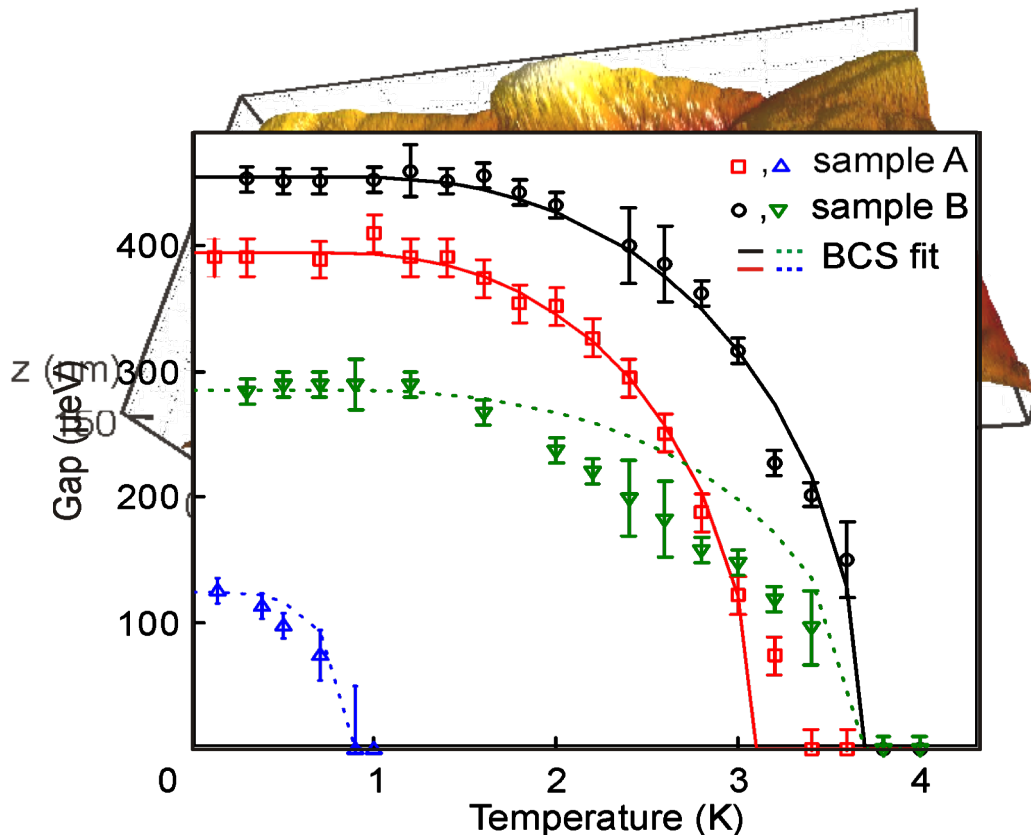
Randomly oriented small grains nucleated on the seeds,

Diamonds seeds (nm size).

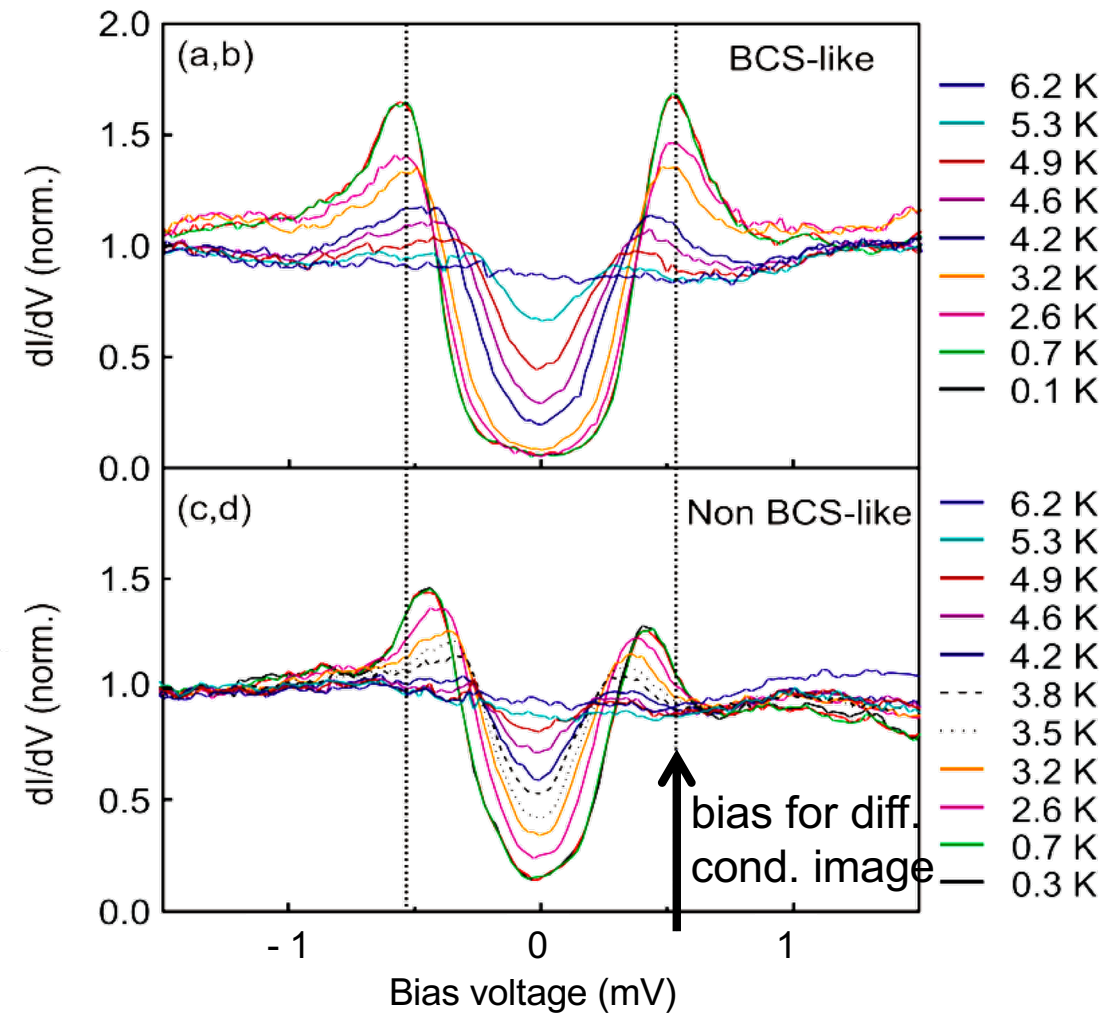
# Poly-BDD tunneling spectroscopy

## Topography

Granular morphology:  
average height 80 nm,  
size less than 150 nm

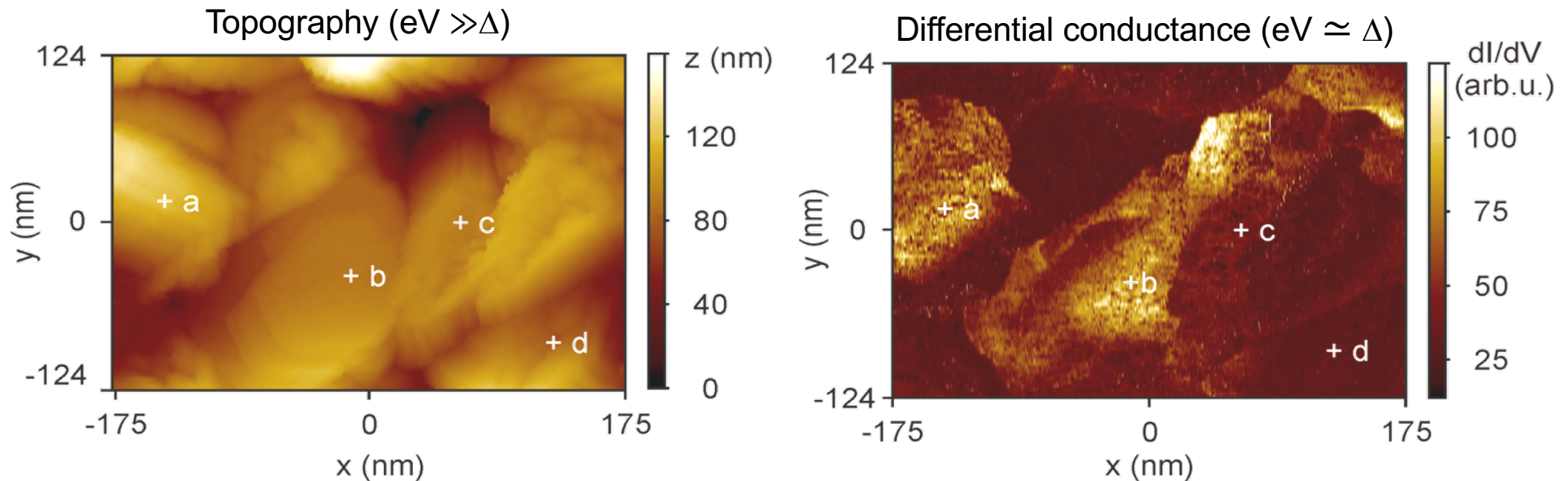


## Spectroscopy



Different electrical properties: BCS (a,b)  
and non-BCS superconductivity (c,d)

# Microstructure superconductivity correlation



Superconducting properties follow the film morphology !

Poly-BDD = disordered network of BCS and non-BCS superconducting grains, proximity effect relevant.

Model system for strongly disordered superconductors.

Applications for innovative superconducting devices.

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## Scanning Tunneling Spectroscopy

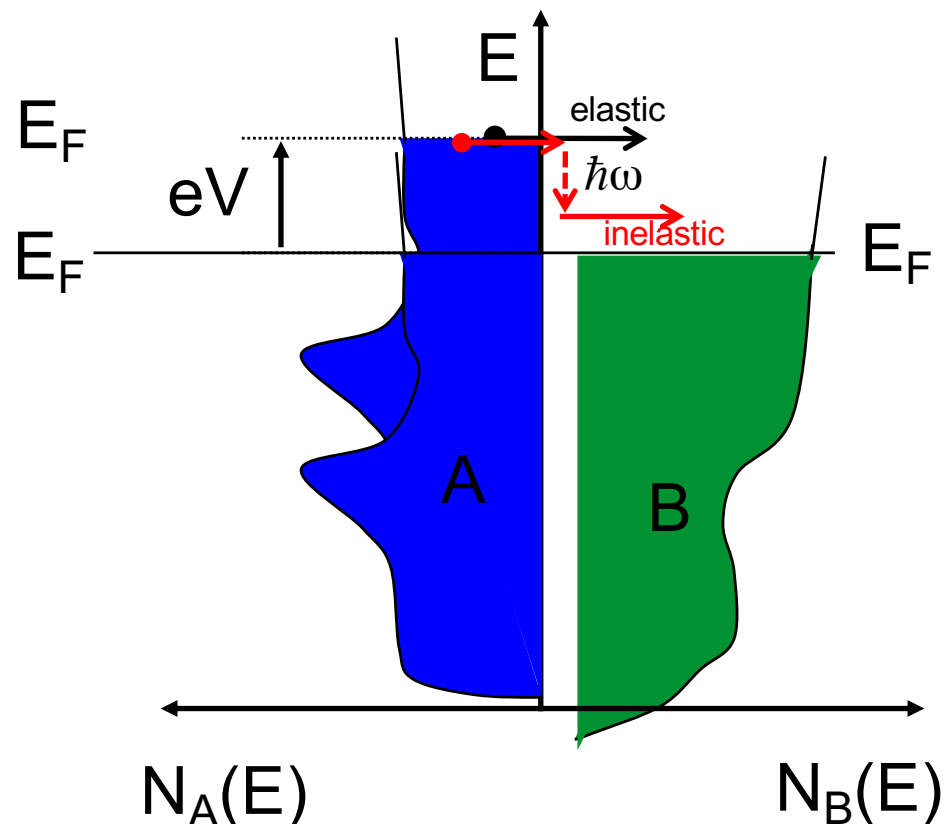
### 4.2 Inelastic Electron Tunneling Spectroscopy (IETS)



# Inelastic tunneling spectroscopy (1)

Tunneling is usually elastic.

An inelastic channel can appear in some cases.



An energy  $\hbar\omega$  is emitted as a phonon or a photon.

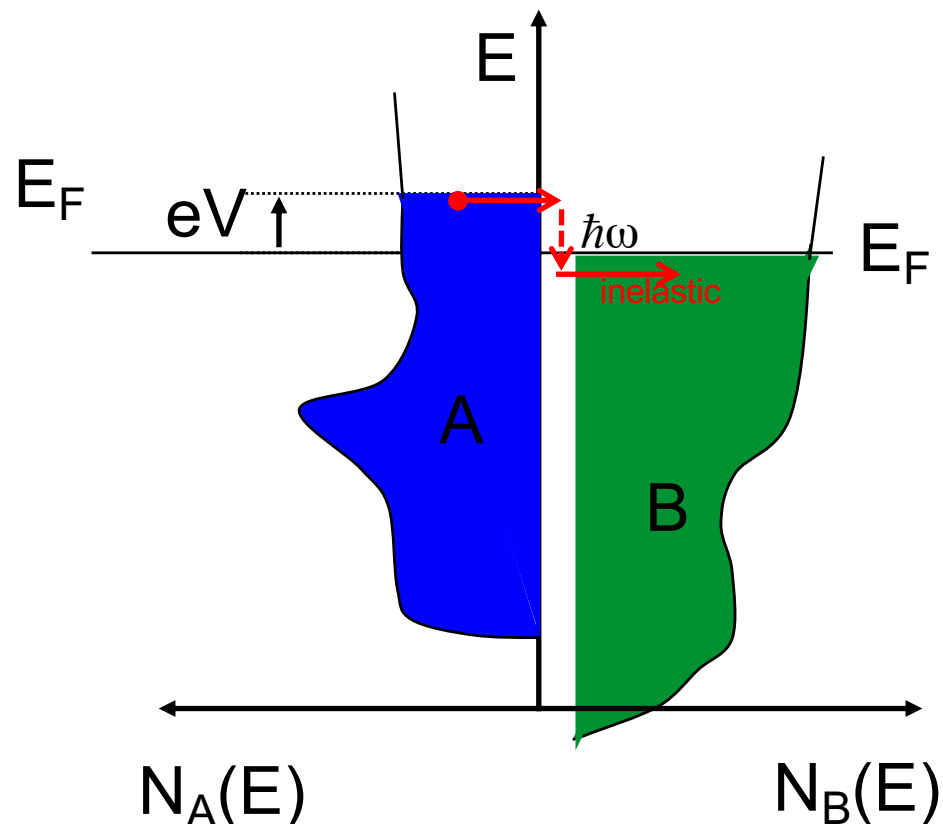
Threshold defined by the excitation energy.

$$|eV| > \hbar\omega$$

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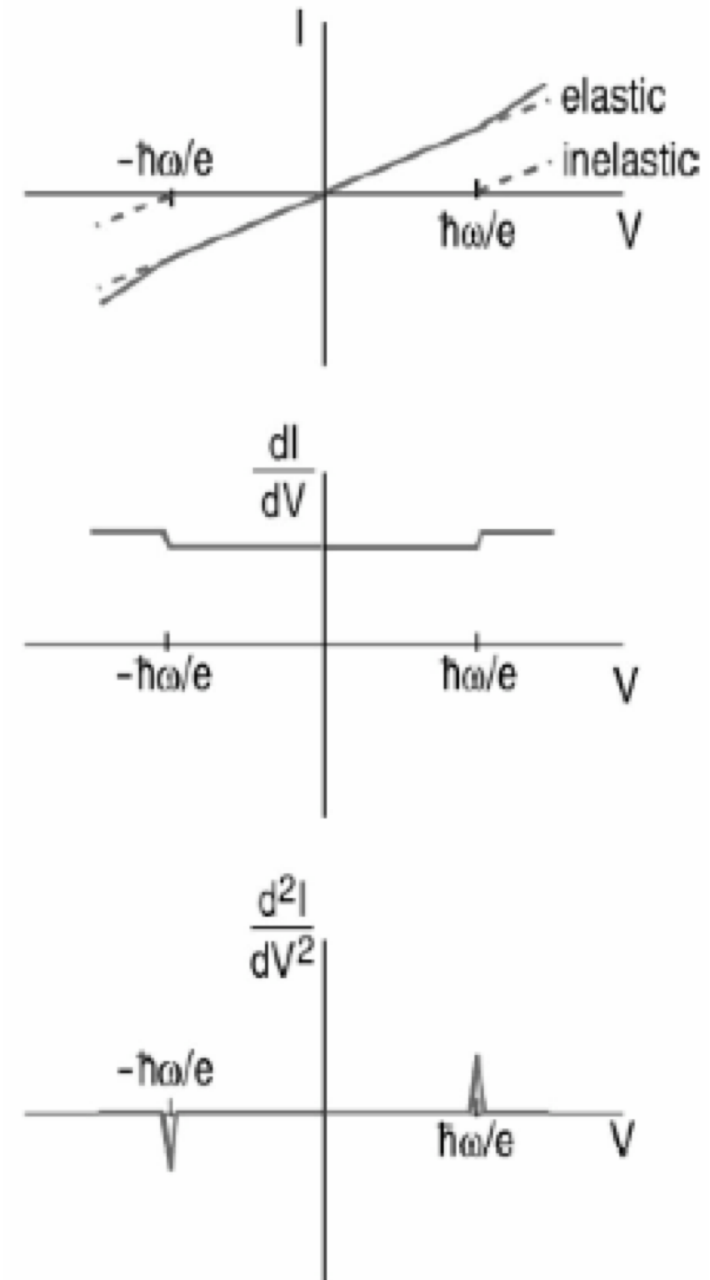
Threshold defined by the excitation energy.

$$|eV| > \hbar\omega$$

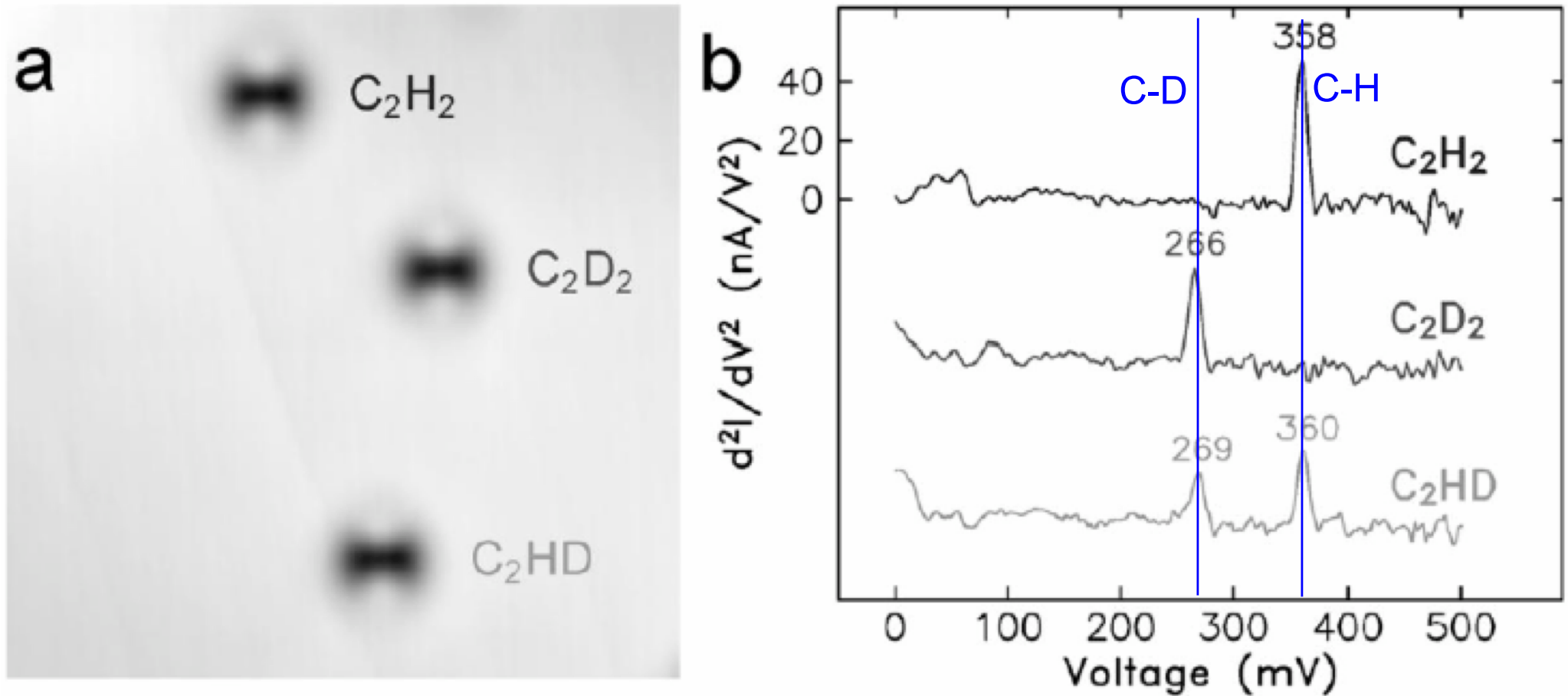
# Inelastic tunneling spectroscopy (2)

Change in current too small to be measured in I-V. More easily accessed in second derivative.

W. Ho, J. Chem. Phys. 117, 11033 (2002)



# Inelastic tunneling spectroscopy (3)



Isotopes can be distinguished: single bond sensitivity of STM-IETS.

B.C. Stipe, M.A. Rezaei, and W. Ho, Science 280, 1732 (1998).

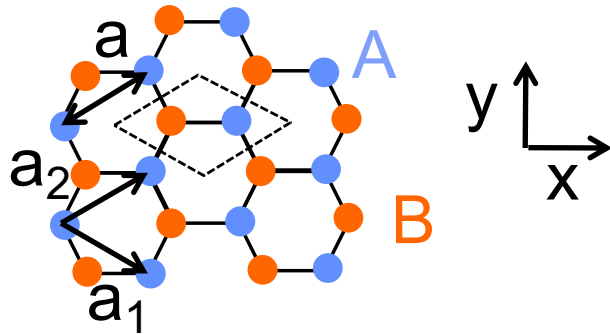
56 x 56 Å<sup>2</sup> STM image and STM-IETS spectra of acetylene isotopes on Cu(001), 8 K. Bright protrusions are the C-H,D bonds, the central depression dark is the C-C bond.

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## Scanning Tunneling Spectroscopy

### 4.3 Graphene

# Electronic states in graphene (1)



Two atoms per unit cell (diamond),  
A and B.  
 $a = 0.246 \text{ nm}$

$sp^2$ :  $s$ ,  $p_x$  and  $p_y$  hybridize to form  $\sigma$  states, which are in-plane, occupied by 3 electrons.  $p_z$  states allow conduction: to be considered in the following, occupied by one electron per atom.

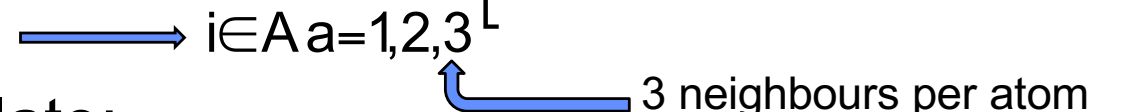
Tight binding approx.: electrons tightly bound to their atom, electronic states are a combination of  $p_z$  atomic orbital.

# Electronic states in graphene (2)

Let's build Bloch states:  $\left| \psi_s^k(\vec{r}) \right\rangle = \frac{1}{\sqrt{N}} \sum_{i \in s} e^{i\vec{k} \cdot \vec{r}(i)} \left( c_i^s \right)^+ |0\rangle$

Hamiltonian includes hopping between every A (B) site and its neighbors.

$$H = -t \sum_{\text{sub-lattice A} \longrightarrow i \in A} \sum_{a=1,2,3} \left[ \left( c_{i+a}^B \right)^+ c_i^A + \left( c_i^A \right)^+ c_{i+a}^B \right]$$



We calculate:

$$\left\langle \psi_A^k | H | \psi_B^k \right\rangle = -t \sum_{a=1,2,3} e^{i\vec{k} \cdot \vec{r}_a} = t.f(\vec{k}) \quad \text{and} \quad \left\langle \psi_B^k | H | \psi_A^k \right\rangle = t.f^*(\vec{k})$$

Lattice geometry gives:

$$\vec{r}_1 \begin{vmatrix} 0 \\ a/\sqrt{3} \end{vmatrix} \quad \vec{r}_2 \begin{vmatrix} -a/2 \\ -a/2\sqrt{3} \end{vmatrix} \quad \vec{r}_3 \begin{vmatrix} a/2 \\ -a/2\sqrt{3} \end{vmatrix}$$

We have:

$$\left\langle \psi_A^k | H | \psi_A^k \right\rangle = \left\langle \psi_B^k | H | \psi_B^k \right\rangle = 0$$

# Electronic states in graphene (2)

Schrödinger equation writes:

$$-t \begin{pmatrix} 0 & f(\vec{k}) \\ f(\vec{k})^* & 0 \end{pmatrix} \begin{pmatrix} |\psi_A^k\rangle \\ |\psi_B^k\rangle \end{pmatrix} = E(\vec{k}) \begin{pmatrix} |\psi_A^k\rangle \\ |\psi_B^k\rangle \end{pmatrix}$$

Hamiltonian diagonalization gives:  $E(\vec{k}) = \pm t |f(\vec{k})|$

One finds:

$$|f(k)| = \left[ 3 + 2 \cos(k_y a) + 4 \cos(\sqrt{3} k_x a / 2) \cos(k_y a / 2) \right]^{1/2}$$
$$E(k) = \pm t \left[ 3 + 2 \cos(k_y a) + 4 \cos(\sqrt{3} k_x a / 2) \cos(k_y a / 2) \right]^{1/2}$$



# Electronic states in graphene (3)

$$E(\mathbf{k}) = \pm t \left[ 3 + 2 \cos(k_y a) + 4 \cos(\sqrt{3} k_x a / 2) \cos(k_y a / 2) \right]^{1/2}$$

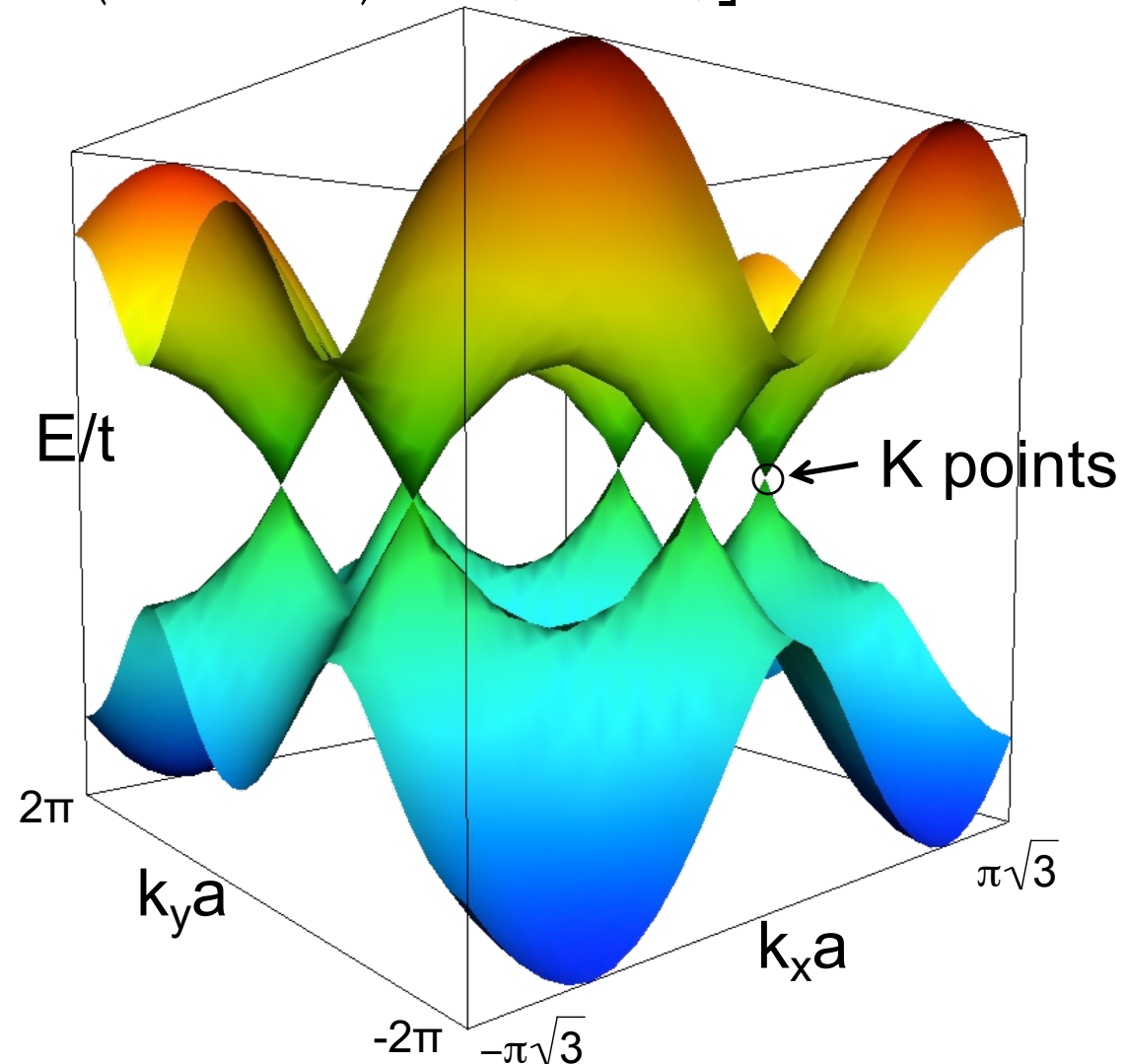
$t = 2.5$  eV (nearest neighbour transfer integral), zero defined at the Fermi level.

1 e<sup>-</sup> / atom in binding orbital  $E < 0$ .

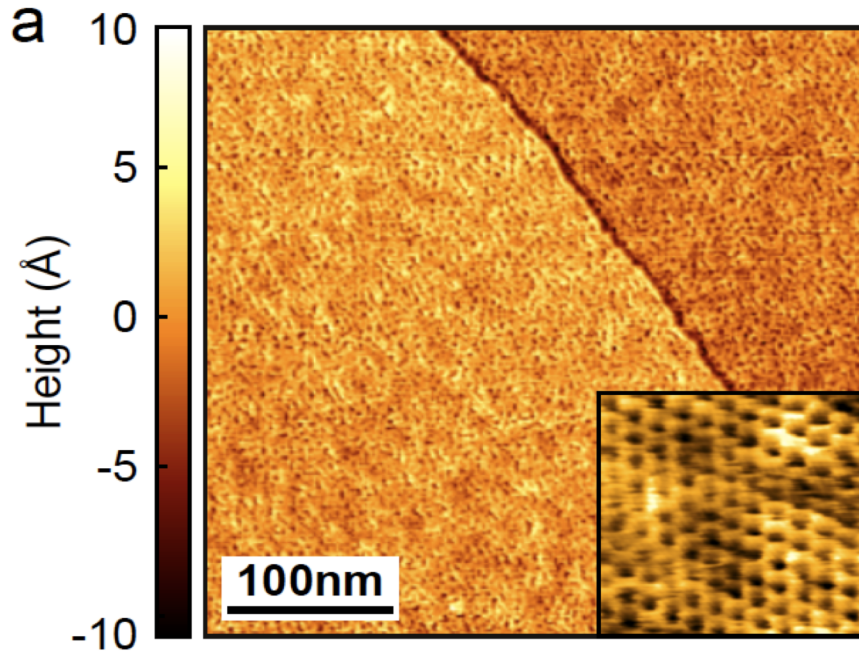
Fermi level made of 6 points K:  
intrinsic graphene is a semi-metal  
or gapless semiconductor.

Close to FL, linear relation  
dispersion: Dirac fermions.

P. R. Wallace, Phys. Rev. 71, 622 (1947).

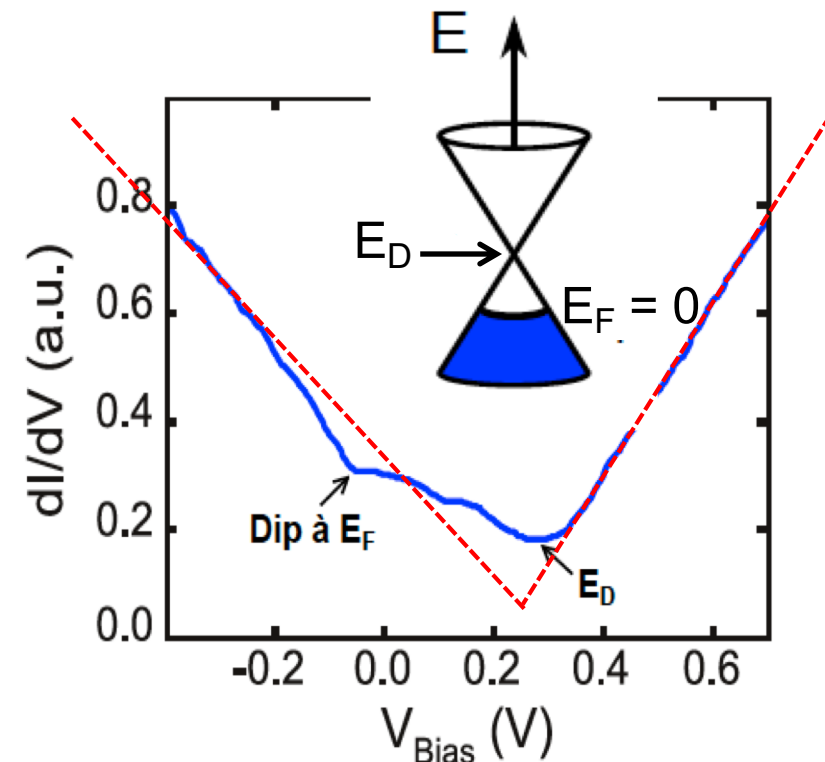


# Topographic disorder and density of states in graphene on Ir (111) after exposure to air

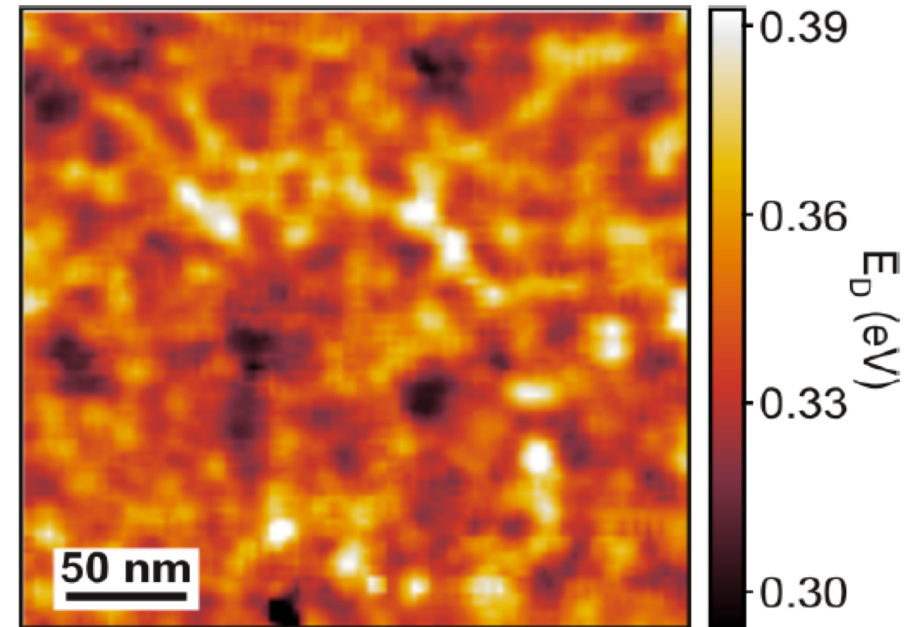
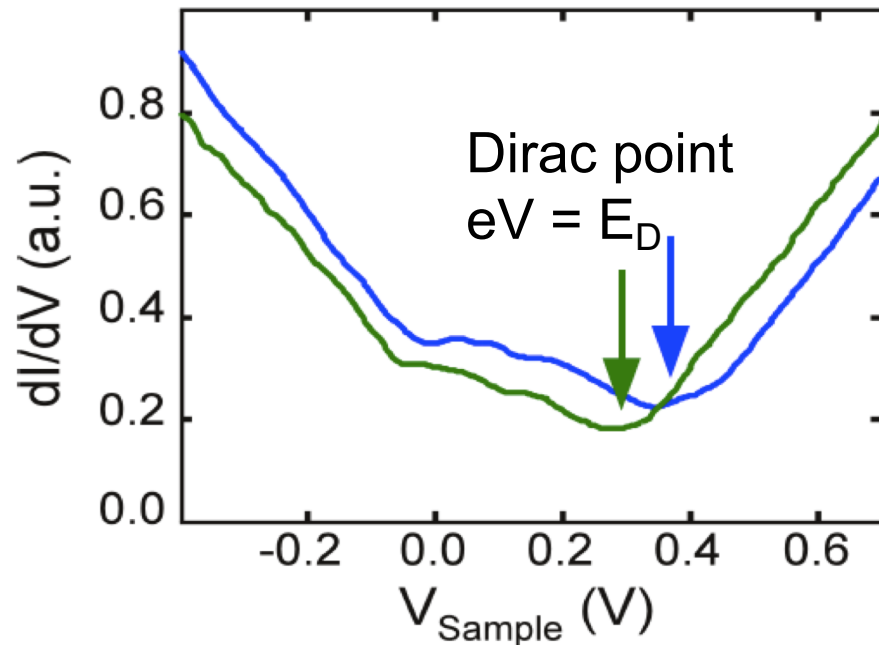


Atomic resolution: clean surface  
No moiré  
Surface roughness  $\sigma_z \approx 100 \text{ pm}$

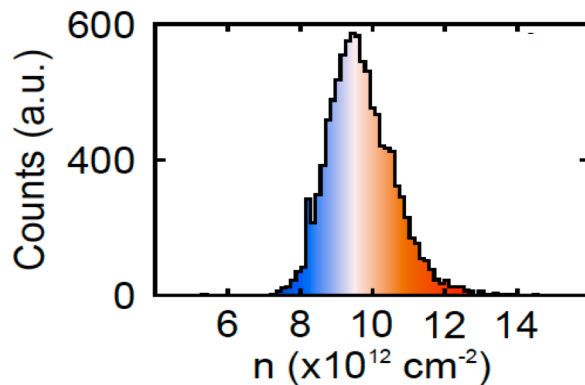
Local density of states is V-shaped  
Minimum at  $eV = E_D$  indicates the **local doping level**, here p type



# Position dependence of the doping



11 000 spectra, ~32h



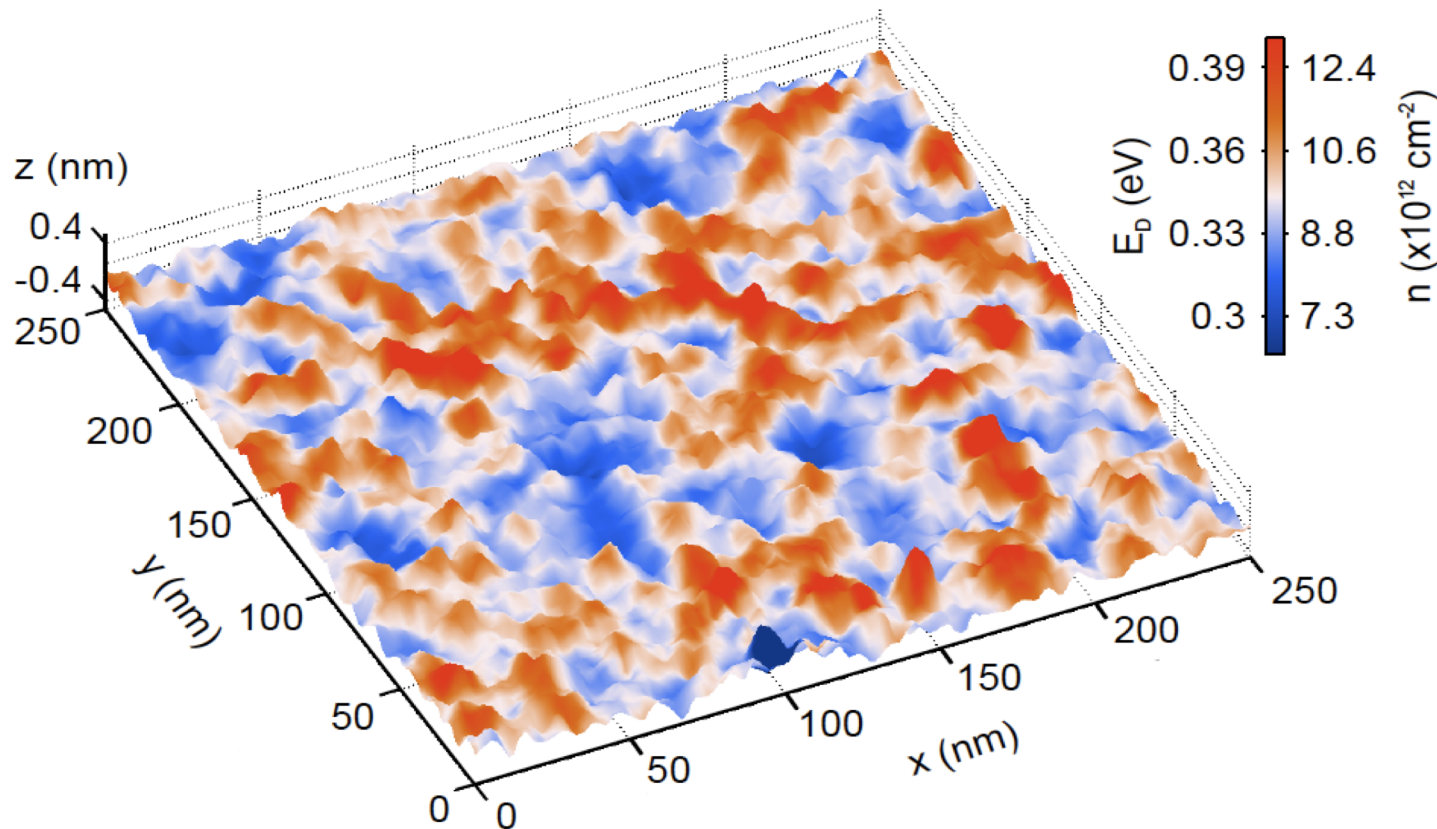
**LDOS maps:** extract  $E_D(\mathbf{r})$ , convert in local doping:

$$E_D = \hbar v_F \sqrt{\pi n}$$

$E_D^0 = 340 \text{ meV}$ ,  $n \approx 10^{13} \text{ cm}^{-2}$ ,  $\sigma_n \approx 10^{12} \text{ cm}^{-2}$ , puddles  $\approx 9 \text{ nm}$ .

Slow method!

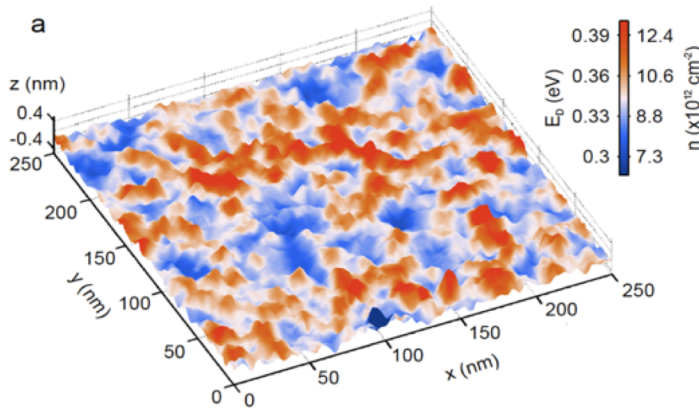
# Differential conductance map



Differential conductance image translated into  $E_D$  map.  
Correlations between topographic height and doping

S. Martin, S. Samaddar, A. Kimouche, J. Coraux, B. Sacépé, H. Courtois, C. Winkelmann, Physical Review B 91, 041406 (2015).

# Calculating a correlation



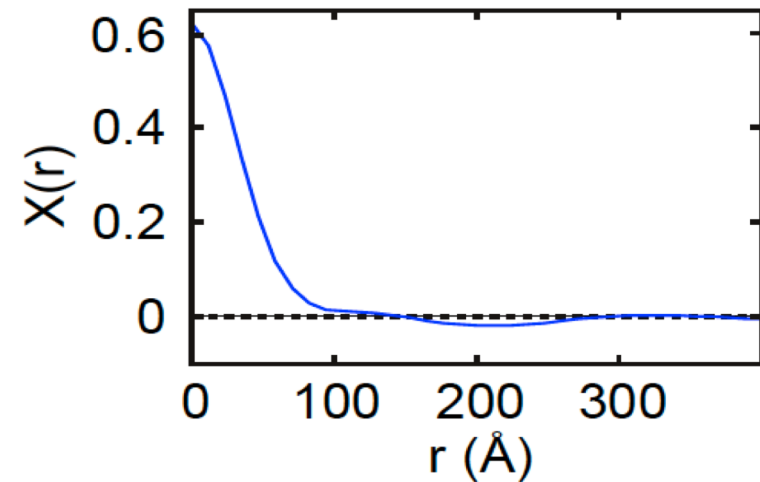
Correlation coefficient as a function of distance:

$$\chi(r) = \frac{\sum_i (E_D(r_i - r) - \bar{E}_D) \cdot (z(r_i) - \bar{z})}{\sigma_{E_D} \cdot \sigma_z}$$

If perfect correlation = 1,  
decorrelated = 0

Strong local correlations between topographic height and doping.

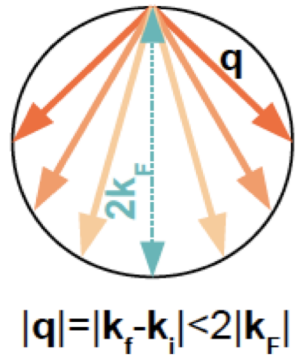
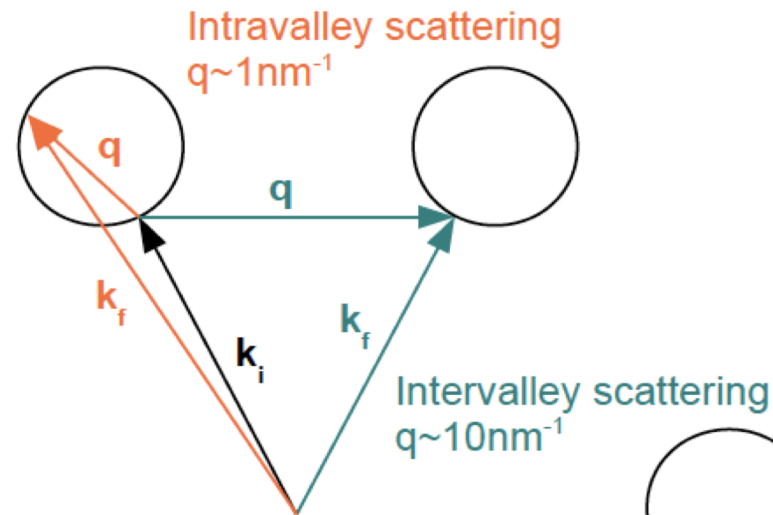
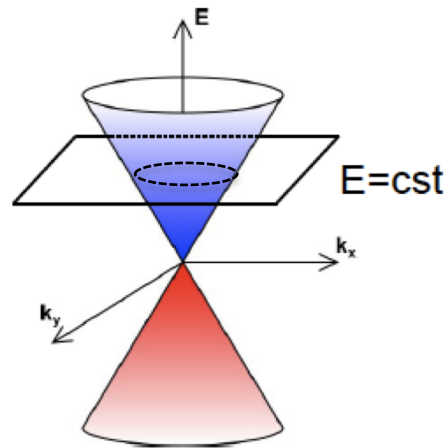
Interpretation: contaminants between Ir and graphene brings doping





# Scattering interferences in graphene

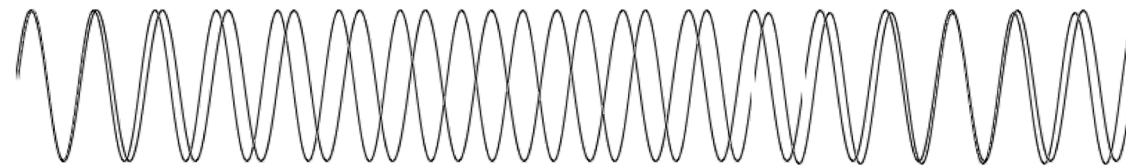
Elastic scattering within graphene:



$$|q| = |k_f - k_i| < 2|k_F|$$

Interference between incident and scattered waves:

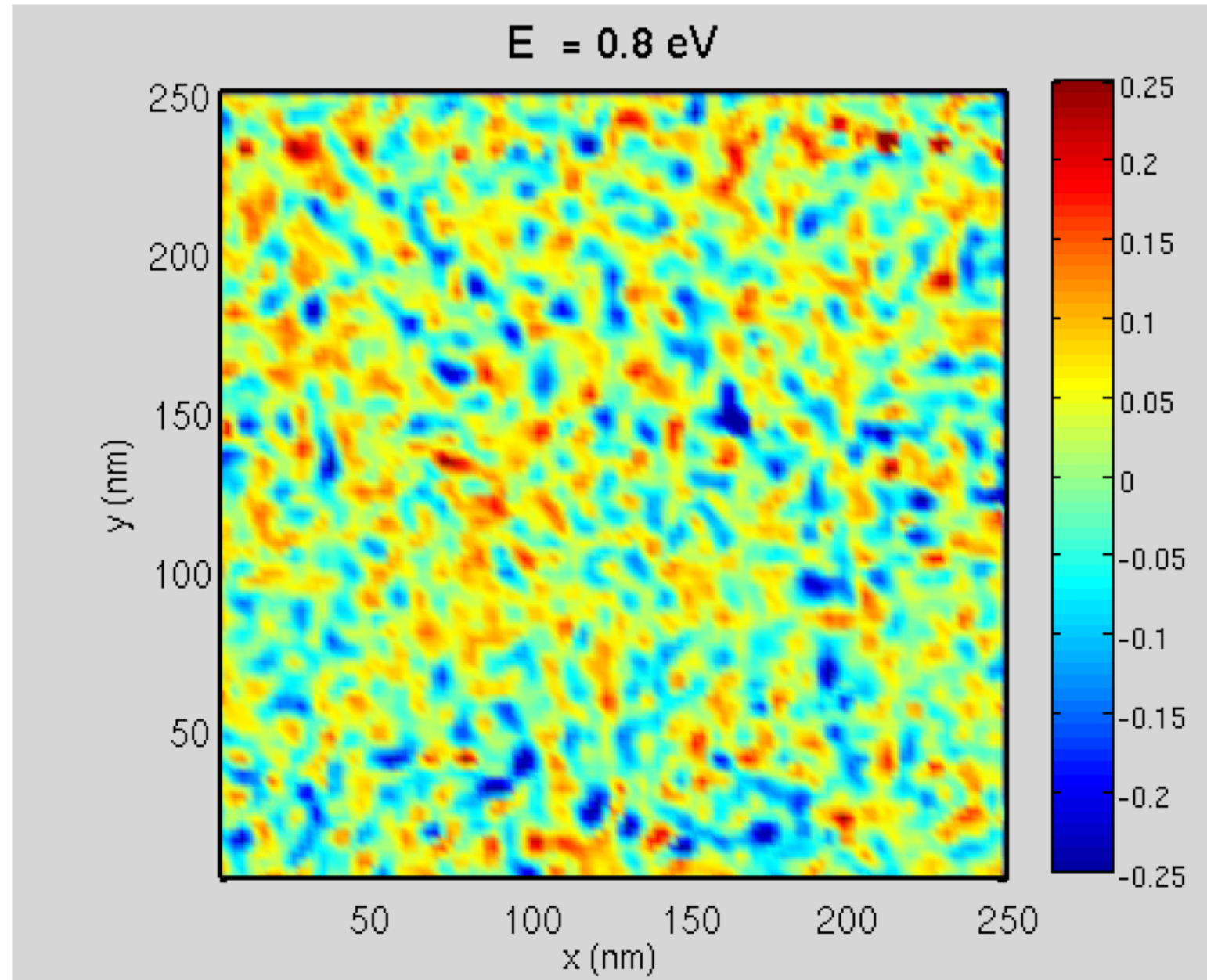
oscillations obtained at the wave-vector difference =  $q$



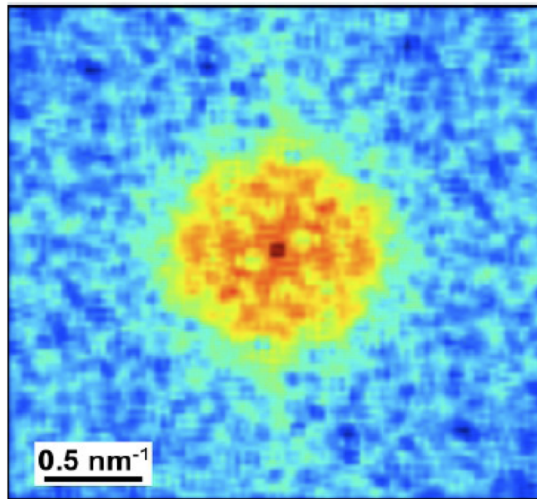
# Density of states inhomogeneities far from $E_D$

Local density of states map at varying energy.

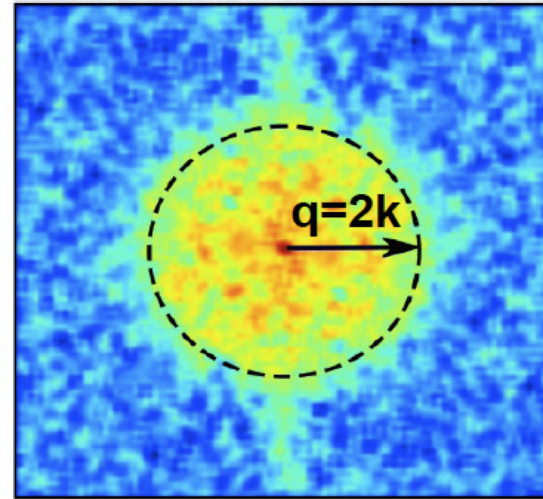
Interferences at a scale smaller than puddles.



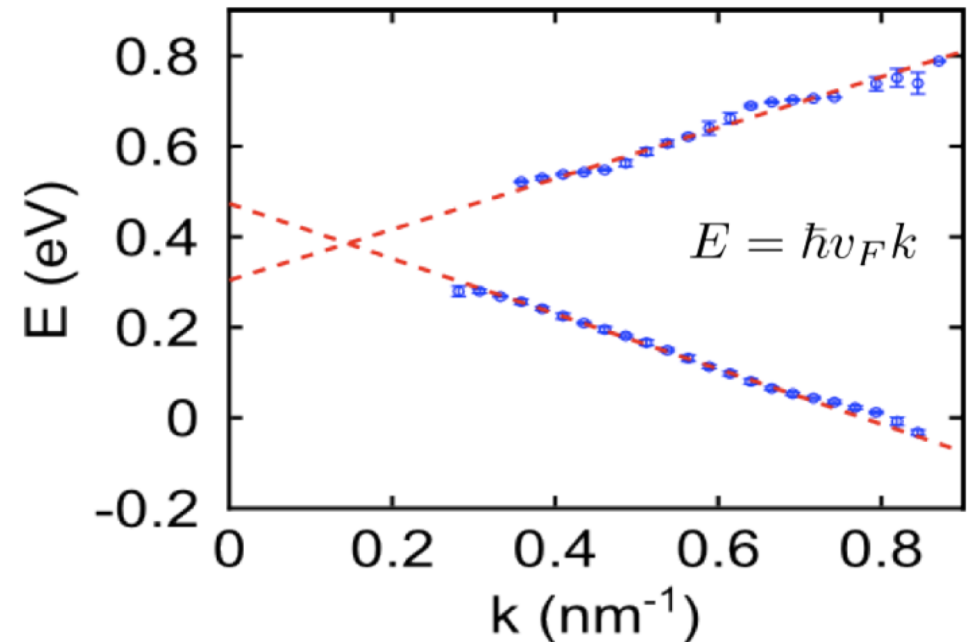
# Fourier transform analysis: intravalley scattering



$E = 15\text{meV}$



125meV



FT of qp interferences images:  
disk (not a ring) with a diameter  $k_F$

Gives access to energy dispersion relation

Linear dispersion relation:  $v_F = 8.9 \pm 0.4 \times 10^5 \text{ m/s}$



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## Scanning Tunneling Spectroscopy

### 4.4 Single-wall carbon nanotubes

# Structure

Discovered in 1991: S. Iijima et al., Nature 354, 56 (1991).

Wrapped graphene sheet with a rolling vector:

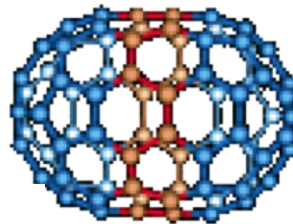
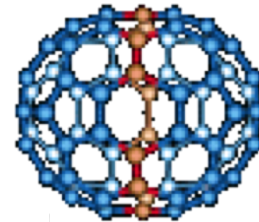
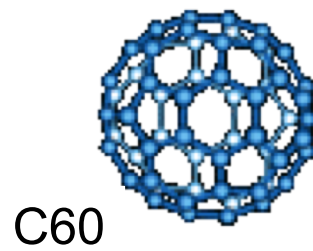
$$\vec{R} = n\vec{a}_1 + m\vec{a}_2$$

(n,m) defines the tube geometry.

$n = m$  : armchair

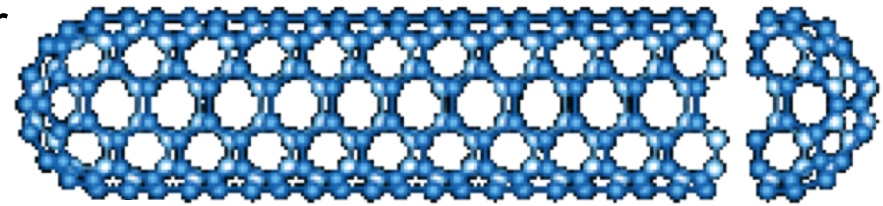
$m = 0$  : zig-zag

$n \neq m$  chiral



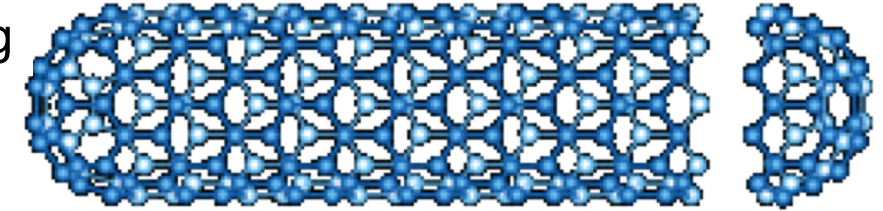
armchair

(5,5)

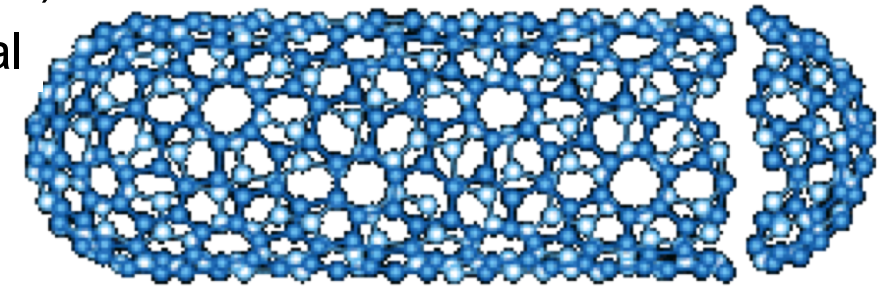


zig-zag

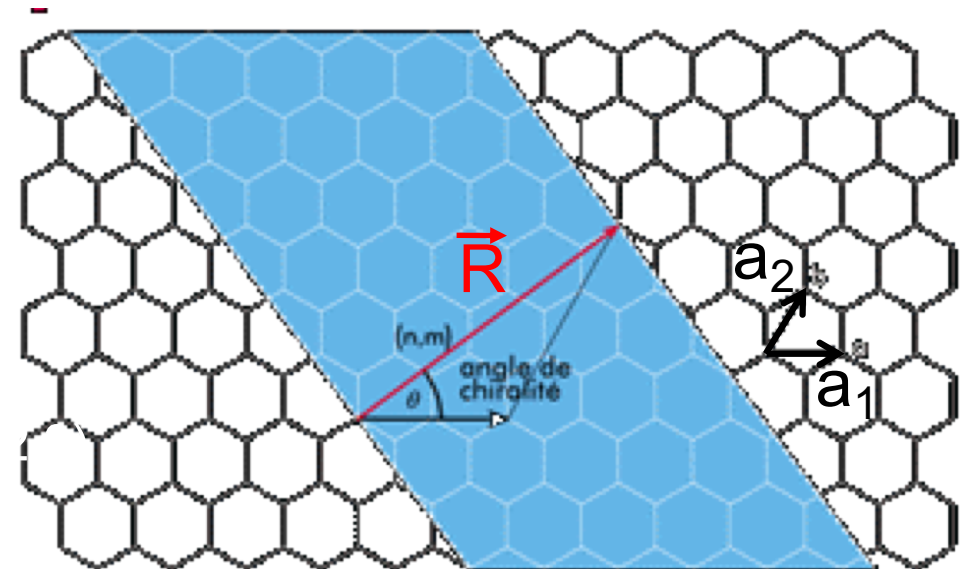
(9,0)



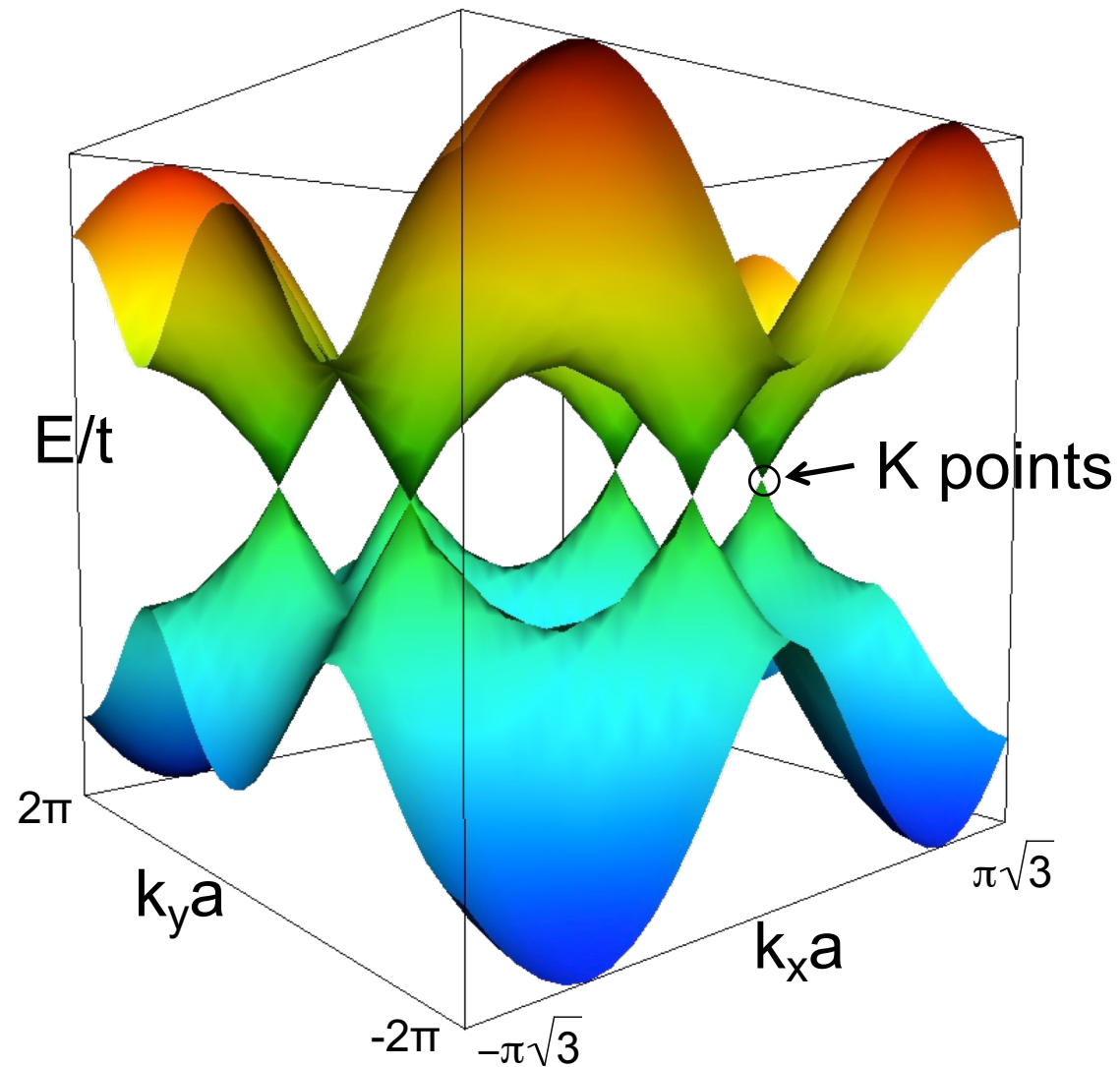
chiral



(10,5)

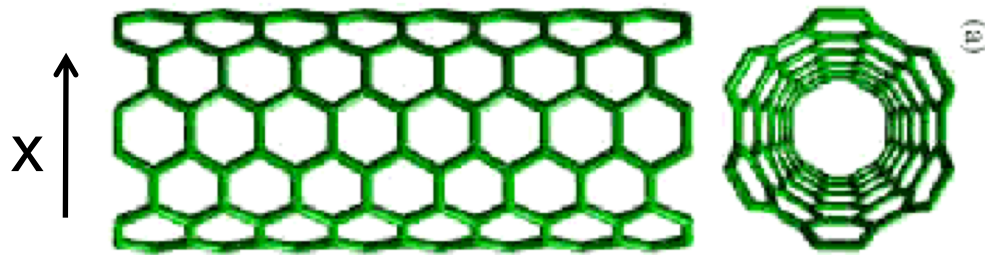


# Electronic states in graphene

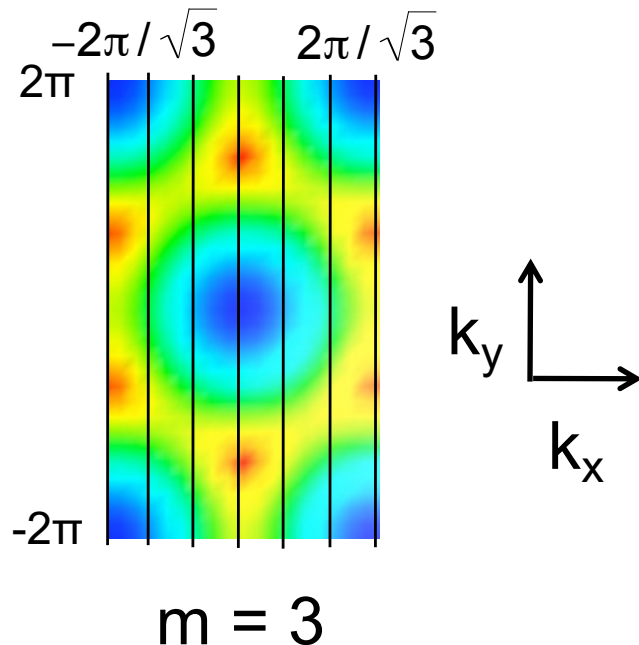


# Armchair tubes are metallic

Armchair, periodic boundary condition around the perimeter:



$$k_x \cdot m\sqrt{3}a = 2\pi p \quad p \text{ integer}$$

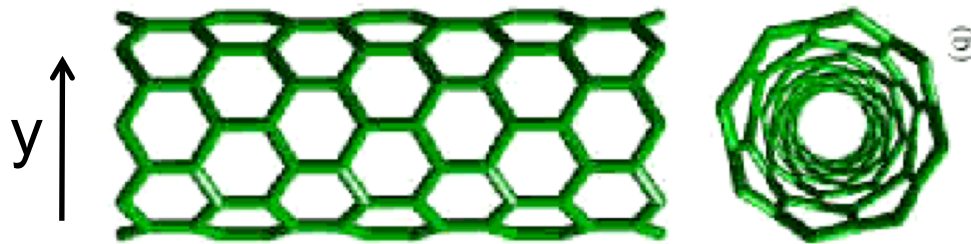


$$k_x a = \frac{2\pi p}{m\sqrt{3}}$$

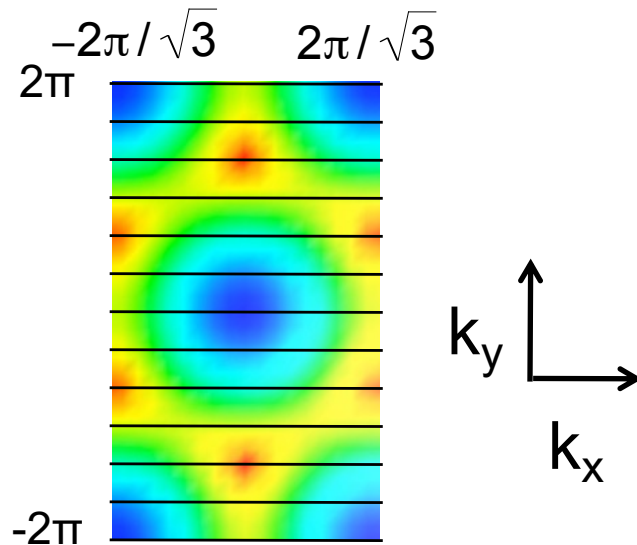
K points in red in this figure.  
Selected k cross K points: metallic.

# Zig-zag can be metallic or semiconducting

Zig-zag, periodic boundary condition around the perimeter:



$$k_y \cdot na = 2\pi p \quad p \text{ integer}$$



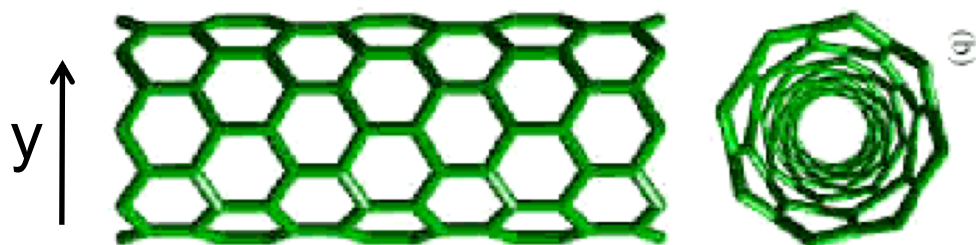
$$k_y a = \frac{2\pi p}{n}$$

K points in red in this figure.  
Selected k cross K points or not,  
depending on n. Metallic if multiple of 3.

$n = 6$ : metallic CNT

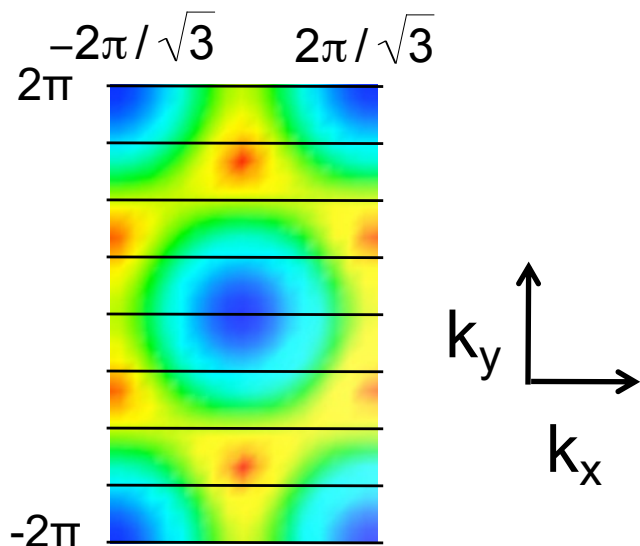
# Zig-zag can be metallic or semiconducting

Zig-zag, periodic boundary condition around the perimeter:



$$k_y \cdot na = 2\pi p$$

$p$  integer



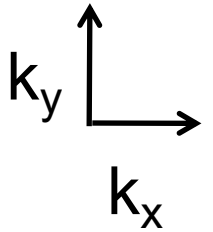
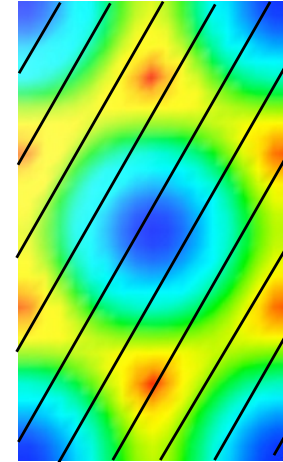
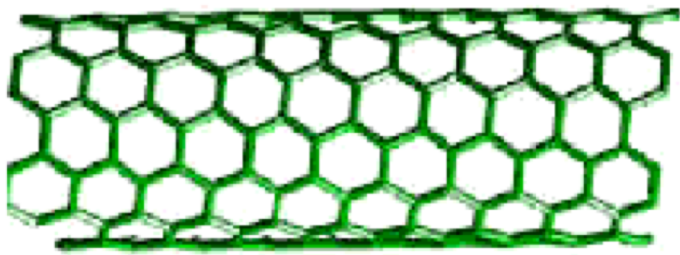
$$k_y a = \frac{2\pi p}{n}$$

K points in red in this figure.  
Selected  $k_y$  cross K points or not,  
depending on  $n$ . Metallic if multiple of 3.

$n = 4$ : semiconducting CNT



# Chiral nanotubes ...



Boundary condition:

$$k_x \cdot m\sqrt{3}a + k_y \cdot (n - m)a = 2\pi p$$

Selected  $k_x, k_y$  lines can cross K points, depending on  $(m, n)$ , in which case the CNT is metallic.

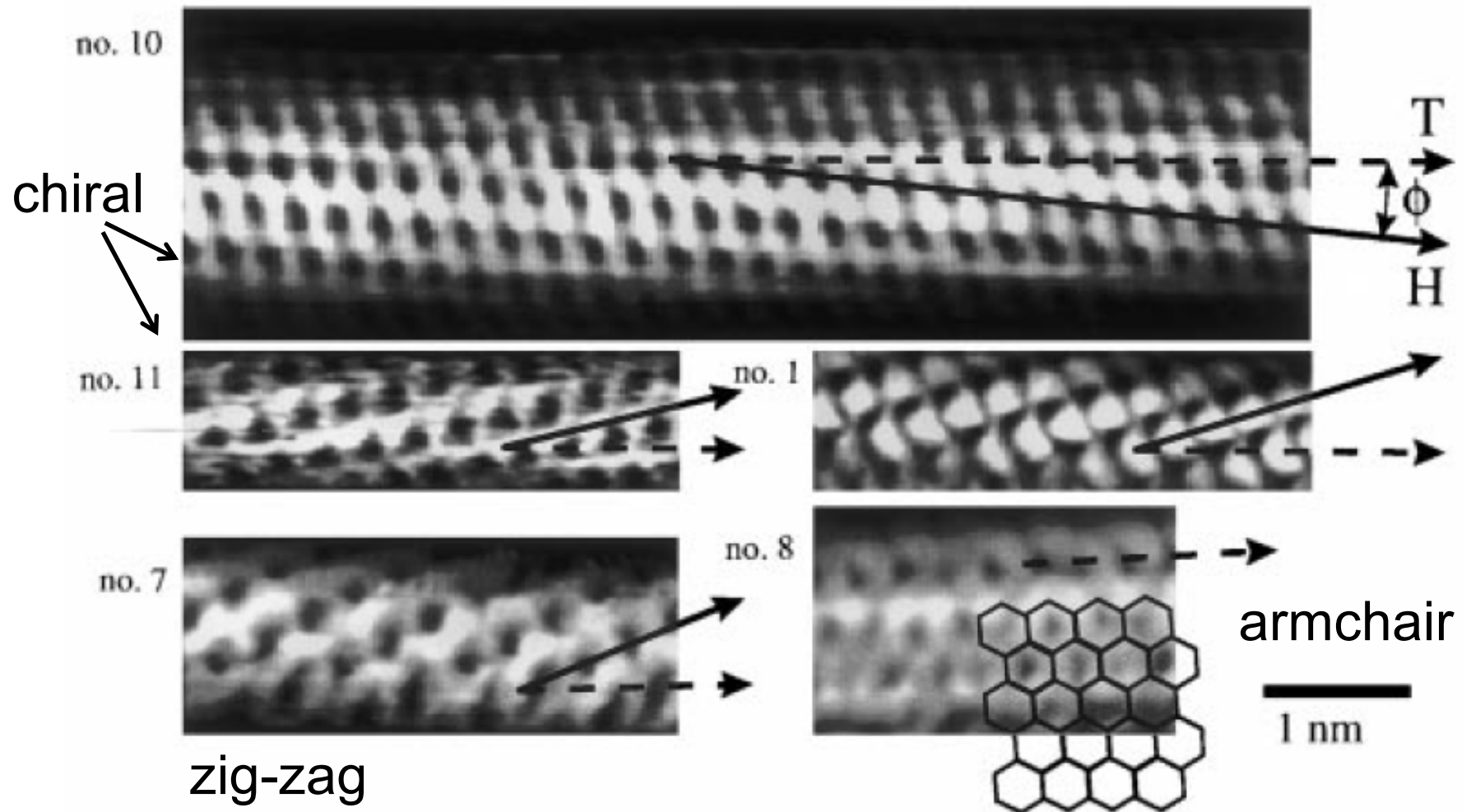
Theoretical prediction:

$n - m = 3k$  : metallic,

$n - m \neq 3k$  : semiconductor.

# CNT imaging

CNTs  
on a Au  
surface.



J.W.G. Wildoer, C. Dekker et al, Nature 391, 59 (1998).



# CNT spectroscopy

Metallic and semiconducting tubes identified.

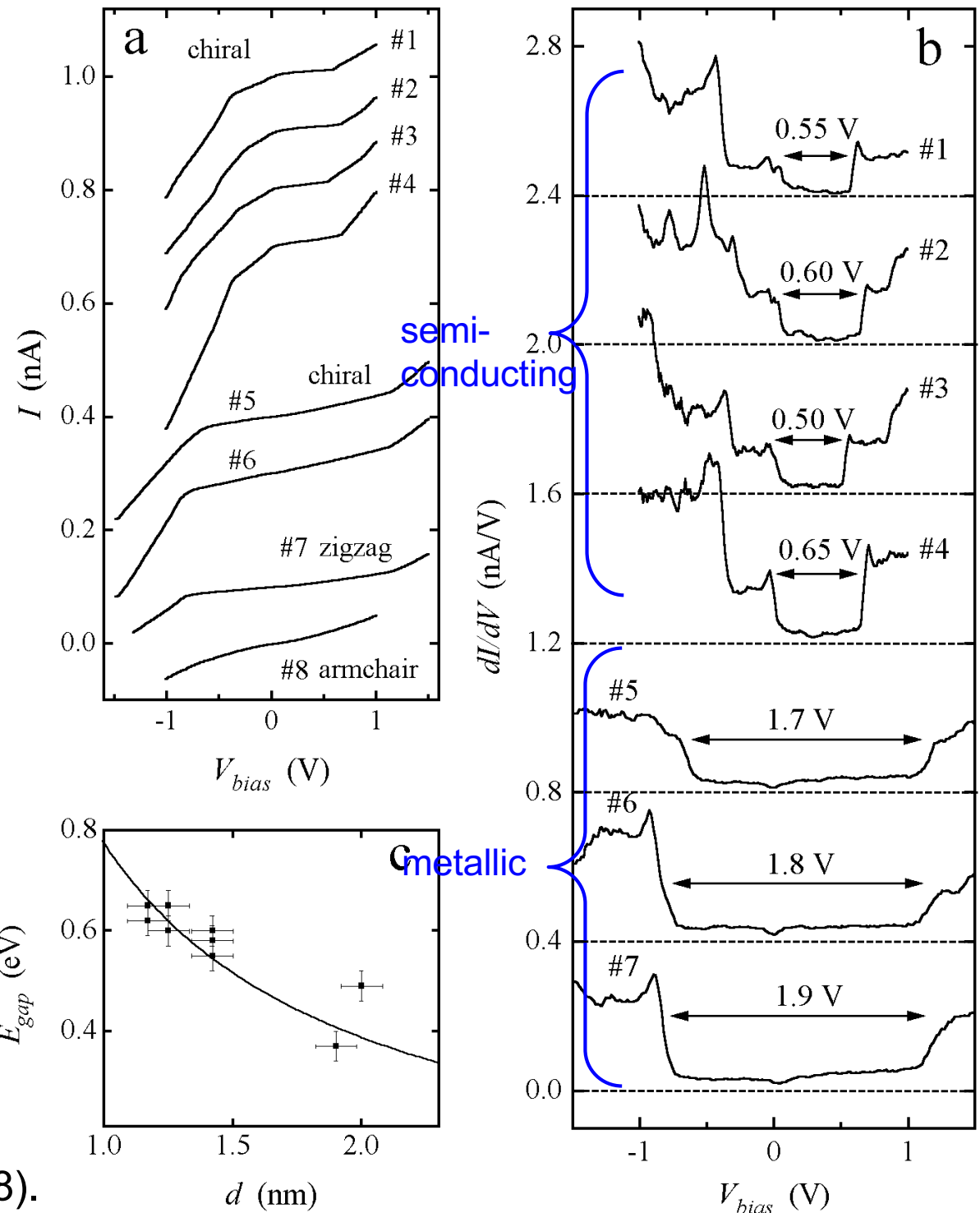
Armchair tubes found metallic. Statistics agrees with 1/3 of chiral ones being metallic.

Energy gap in SC tubes:

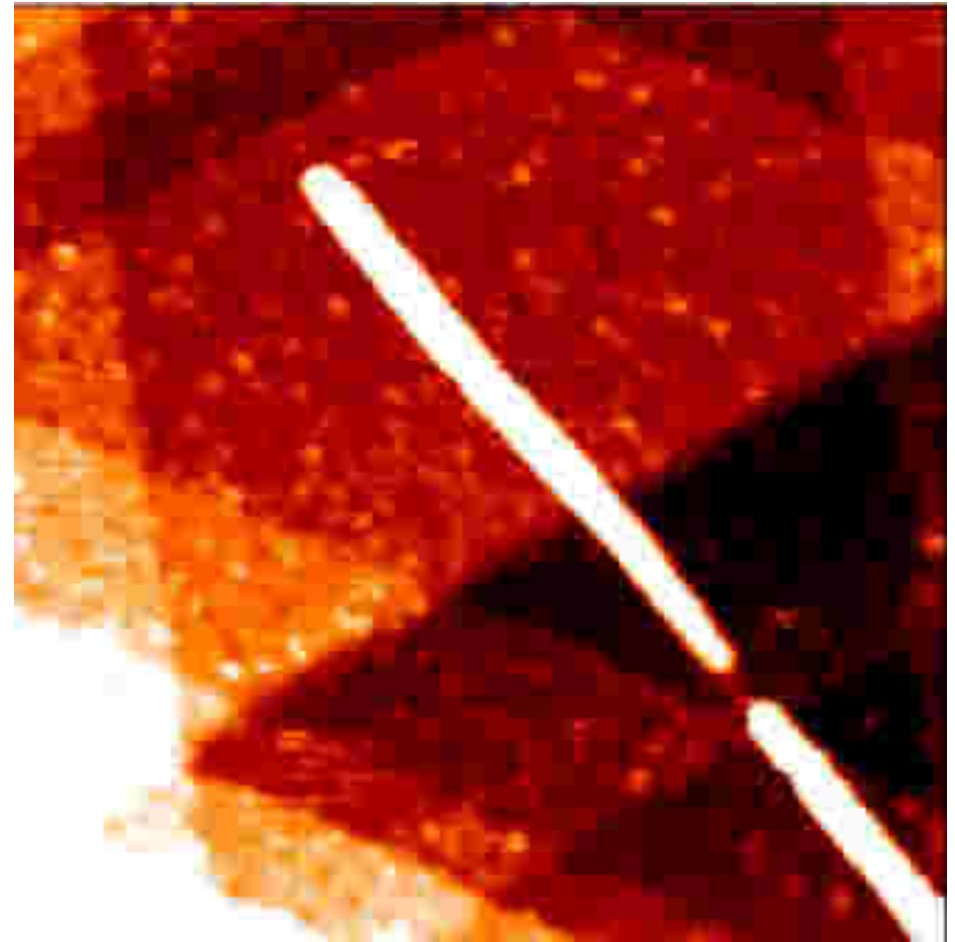
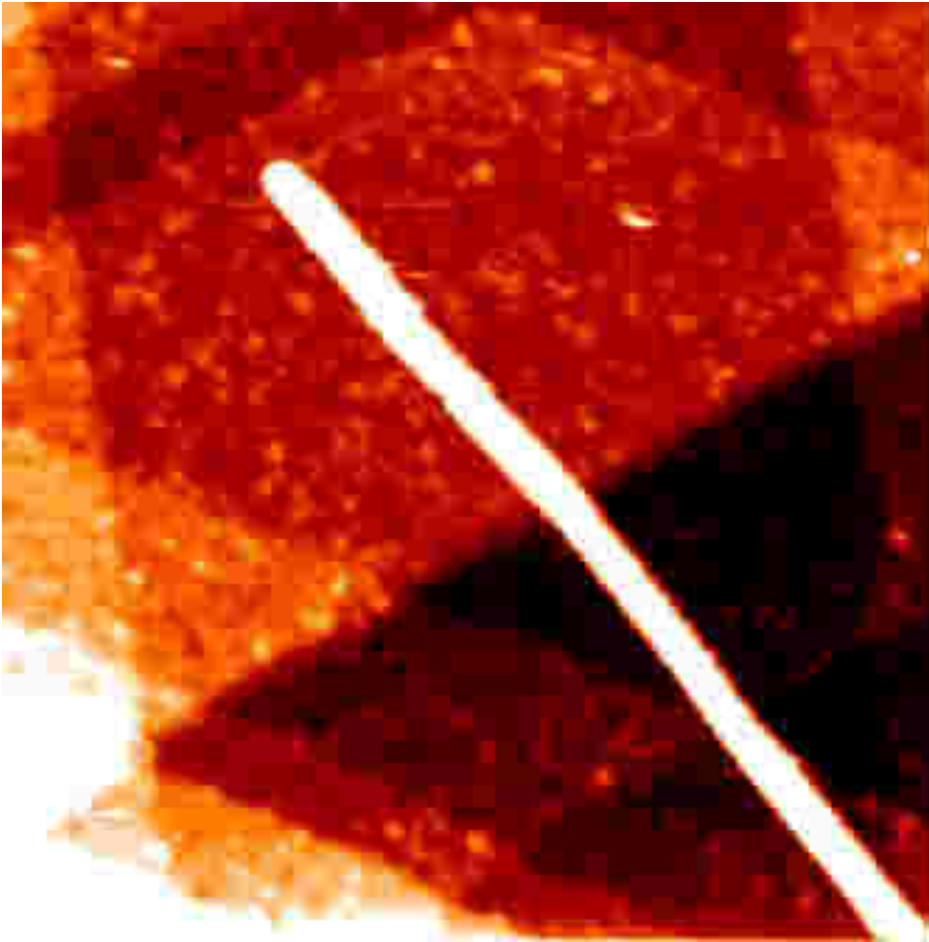
$$E_{\text{gap}} = 2\gamma a / \sqrt{3}d$$

d: tube diameter.

J.W.G. Wildoer et al, Nature 391, 59 (1998).



# Cutting a nanotube



Voltage pulse (5 V) in the STM mode.

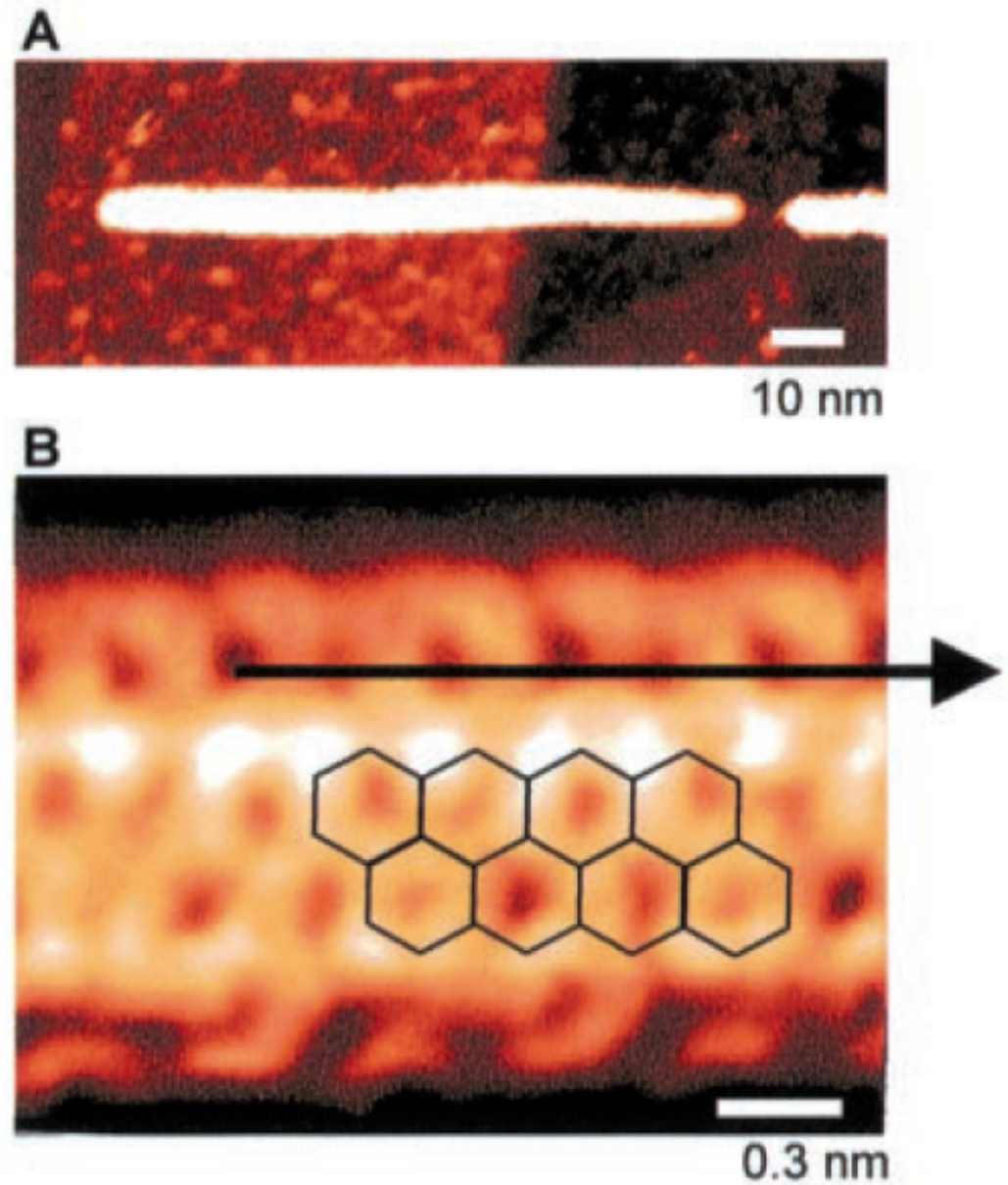
L. C. Venema, J. W. G. Wildöer, H. L. J. Temminck Tuinstra, C. Dekker, A. G. Rinzler and R. E. Smalley, Appl. Phys. Lett. 71, 2629 (1997).

# Imaging electron wave functions in a CNT (1)

Armchair (metallic) tube cut with a voltage pulse:

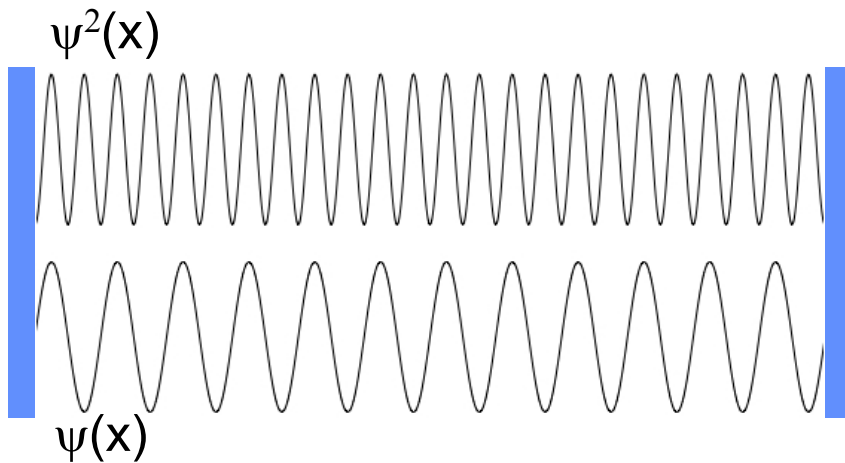
Tunnel contact  $I(V)$  and  $dI/dV$  evolves spatially at low energy.

LDOS spatial modulation.

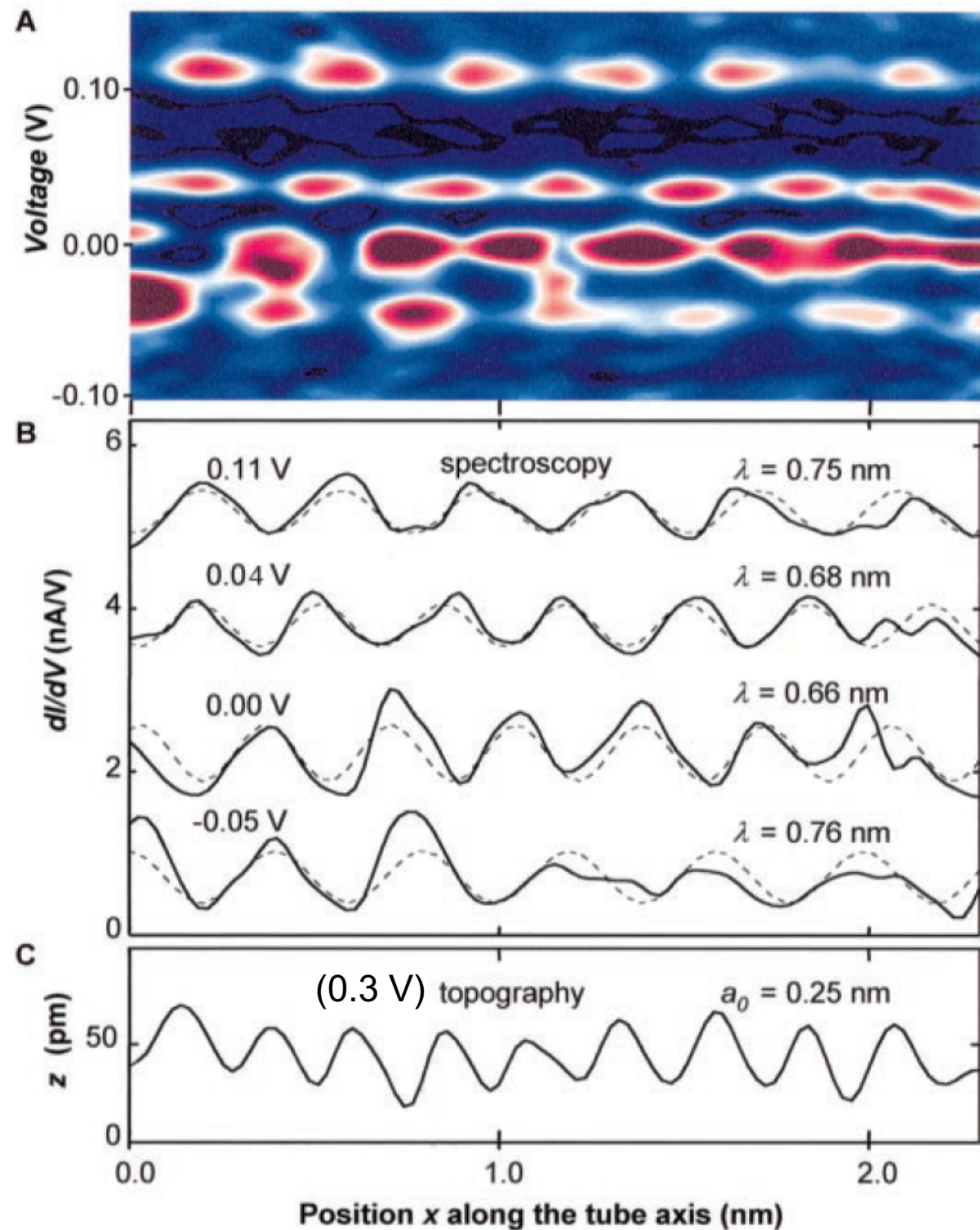


# Imaging electron wave functions in a CNT (2)

Period different from atomic cell size, close to Fermi wavelength: electronic standing waves.



$$\psi(x) = A \sin(2\pi x/\lambda)$$





# Imaging electron wave functions in a CNT (2)

Differential conductance proportional to electron density:

$$\text{Fit: } \frac{dI}{dV} = G_1 \sin^2(2\pi x/\lambda) + G_0$$

Textbook model of a particle in a 1D box, here about 100 e-.

